Minimum sets of plurisubharmonic functions

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A set $K \subset \mathbb{C}^n$ is called the minimum set of a real valued function f if $f_{|K} = \min\{f(x) : x \in D_f\}$ and $f(x) > f_{|K}$ for any $x \in D_f \setminus K$. In this talk we will try to address the problem how the minimum sets of plurisubharmonic functions look like under certain additional assumptions.

A classical theorem in this area is the following one by Harvey and Wells ([HW73]): The zero set of a nonnegative strongly plurisubharmonic function (of class $\mathcal{C}^2$) is contained in a totally-real submanifold of class $\mathcal{C}^1$ of the domain of definition of the function. In particular it follows that the Hausdorff dimension of this set is small (literally: does not exceed n) and that this set does not have an analytic structure.

Building on this theorem in my papers [DD16],[DD17](with Sławomir Dinew) we studied how the assertion changes when we drop the regularity assumption or assume just that the Monge-Ampère measure is positive.

In the tall I will show our results: K may contain analytic subsets, and its Hausdorff dimension can be (a bit) greater than n. After that I will show examples of fractal sets which are minimum sets.

References

[DD16] S. Dinew, Ż. Dinew: The minimum sets and free boundaries of strictly plurisubharmonic functions, Calc. Var. Partial Differential Equations **55**(6) (2016), Article:148,http://link.springer.com/article/10.1007/s00526-016-1069-5.

[DD17] S. Dinew, Ż. Dinew: Differential tests for plurisubharmonic functions and Koch curves, https://arxiv.org/abs/1607.00893

[HW73] F.R.Harvey, R.O.Wells Jr.: Zero sets of non-negative strictly plurisubharmonic functions. Math. Ann. **201** (1973), 165-170, http://link.springer.com/article/10.1007/BF01359794.