DONALDSON'S EQUATION IN $\mathbb{C}^2$

SZYMON MYGA

Following Donaldson we introduce the Riemannian structure on the space of volume forms on compact manifold X. The geodesic equation in this space is called Donaldson's equation. For a function $u : [0,1] \times X \ni (t,x) \to \mathbb{R}$ such that $1 - \Delta_x u > 0$, the Donaldson's equation is

$$\ddot{u} = \frac{|\nabla_x \dot{u}|^2}{1 - \Delta_x u},$$

where dot indicates time derivative.

We are interested in its nonlinear part, i.e. $Q(D^2u) = \ddot{u}\Delta_x u - |\nabla_x \dot{u}|^2$ (change of sign before Δ_x is for convinience, it doesn't affect the equation). We notice that in $\mathbb{R}^3$ the solution to equation $Q(D^2u) = f$ can be extended to $\mathbb{C}^2$, as a solution to Monge-Ampere equation that is independent of one imaginary parameter.

Thus in $\mathbb{R}^3$, having a solution $Q(D^2u) = 1$ that is stricly convex in time variable and stricly subharmonic in space variable gives also a solution to unimodular Monge-Ampere equation that is stricly plurisubharmonic and allows us to construct Kähler metrics on $\mathbb{C}^2$.

For the rest of the talk we talk about Kähler metrics that are given by known solutions to $Q(D^2u) = 1$ in $\mathbb{R}^3$.