

A REMARK ON THE ALEXANDROV–FENCHEL INEQUALITY

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Summary. We shall first recall a few notions in convex geometry: Minkowski sum, Legendre transform, gradient map, mixed volume, etc. Our main result is a complex-geometric proof of the Alexandrov–Fenchel inequality without using toric compactifications. The idea is to use the Legendre transform and develop the Brascamp–Lieb proof of the Prékopa theorem. New ingredients in our proof include an integration of Timorin’s mixed Hodge–Riemann bilinear relation and a mixed norm version of Hörmander’s L^2 -estimate, which also implies a non-compact version of the Khovanskii–Teissier inequality.