m-SUBHARMONIC FUNCTIONS ON COMPACT SETS

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Let $\mathcal{SH}_m^o(X)$ denote the set of functions that are the restriction to X of functions that are m-subharmonic and continuous on some neighborhood of $X \subseteq \mathbb{C}^n$.

Next, we define a class of Jensen measures.

Definition 1. Let X be a compact set in $\mathbb{C}^n$, $1 \leq m \leq n$, and let μ be a non-negative regular Borel measure defined on X with $\mu(X) = 1$. We say that μ is a *Jensen measure with barycenter* $z \in X$ if

$$u(z) \leq \int_X u\, d\mu \qquad \text{for all } u \in \mathcal{SH}_m^o(X)\,.$$

The set of such measures will be denoted by $\mathcal{J}_z^m(X)$.

With the help of the Jensen measures we can define m-subharmonic functions on compact sets.

Definition 2. Let X be a compact set in $\mathbb{C}^n$. An upper semicontinuous function u defined on X is said to be *m-subharmonic on* X, $1 \leq m \leq n$, if

$$u(z) \leq \int_X u\, d\mu\,, \quad \text{for all } \ z \in X \ \text{ and all } \ \mu \in \mathcal{J}_z^m(X)\,.$$

The set of m-subharmonic functions defined on X will be denoted by $\mathcal{SH}_m(X)$. A function $h : X \to \mathbb{R}$ is called *m-harmonic* if h, and $-h$, are m-subharmonic. The set of all m-harmonic functions defined on X will be denoted by $\mathcal{H}_m(X)$.

Definition 3. Let $1 \leq m \leq n$, and let X be a compact set in $\mathbb{C}^n$. The *Choquet boundary* of X w.r.t. $\mathcal{J}_{z_0}^m$ is defined as

$$O_X^m = \left\{ z \in X : \mathcal{J}_{z_0}^m = \{\delta_z\} \right\}.$$

The *Šilov boundary, B_X^m, of* X is defined to be the topological closure of O_X^m.

We shall prove the characterization of Šilov boundary by peak m-subharmonic functions and by harmonic measures.

We characterize those compact sets X for which the Dirichlet problem has a solution within the class of continuous m-subharmonic or m-harmonic functions defined on a compact set, i.e. for any $f \in \mathcal{C}(B_X^m)$ one can find $u \in \mathcal{SH}_m(X) \cap \mathcal{C}(X)(\mathcal{H}_m(X) \cap \mathcal{C}(X))$ such that $u = f$ on B_X^m.