# On maximum principles

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For $\Omega \subseteq \mathbb{R}^n$-open and $p \in \Omega$ - stationary point of $f : \Omega \to \mathbb{R}$, $\quad (\nabla f(p) = 0)$ the "classical" second order optimality conditions read:

[Second Partial Derivative Test - sufficient condition] If $f$ is twice (Fréchet) differentiable at $p$ then a sufficient condition for the existence of a strong local extremum is

- Positive definiteness of the Hessian matrix of $f$ at $p$ $(Hess_f(p) > 0) \Rightarrow$ $f$ has a strong local minimum at $p$.
- Negative definiteness of the Hessian matrix of $f$ at $p$ $(Hess_f(p) < 0)$ $\Rightarrow$ $f$ has a strong local maximum at $p$.

[Second Partial Derivative Test - necessary condition]

If $f$ is twice (Fréchet) differentiable at $p$ then a necessary condition for the existence of a (ordinary) local extremum is

- $f$ has a local minimum at $p \Rightarrow$ nonnegative definiteness of the Hessian matrix of $f$ at $p$ $(Hess_f(p) \geq 0)$.
- $f$ has a local maximum at $p \Rightarrow$ nonpositive definiteness of the Hessian matrix of $f$ at $p$ $(Hess_f(p) \leq 0)$.

In the first part of the talk we investigate how much of the classical second order optimality conditions "can be saved" when $f$ is less regular.

In the second part we try to answer the question how much information do the Hessians of a function near a critical point carry i.e. how much reliable are they in determining that the critical point is a local maximum or local minimum.

These kinds of problems arise in the study of general maximum principles which are themselves utilized in PDE theory (for example in the theory of viscosity solutions).