

Carathéodory balls and proper holomorphic maps on multiply-connected planar domains

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We establish the existence of disconnected open balls and the inequivalence of closed balls and the closure of open balls under the Carathéodory metric in some planar domains of finite connectivity greater than 2. This resolves a problem posed by Jarnicki, Pflug and Vigué in 1992. A corresponding result for some higher dimensional pseudoconvex domains is also obtained. Our results follows from an explicit characterization of proper holomorphic maps from a non-degenerate finitely-connected planar domain onto the unit disk which answers a question posed by Schmieder in 2005. This is analogous to Fatou's famous result that proper holomorphic maps of the unit disk onto itself are finite Blaschke products. A parameter space for proper holomorphic maps is also determined which extends a result of Grunsky in 1941. This is a joint work with Chiu Chak Tang and Jonathan Tsai.