The talk was devoted to the proof of the following theorem due to Kirchheim and Preiss:

Let μ be a regular Borel measure on $\mathbb{R}^n$. Suppose that for every $r > 0$ and every $x, y \in spt(\mu)$ one has

$$\mu(B_r(x)) = \mu(B_r(y)),$$

where $B_r(y)$ denotes the Euclidean ball of radius r centered at y. Then the support of μ is either the whole $\mathbb{R}^n$ or is equal to the zero set of a real-analytic function (and is hence a real-analytic subvariety of $\mathbb{R}^n$).

Examples of such measures are the Lebesgue measure on any affine subspace of $\mathbb{R}^n$. Another nontrivial example is the 3-dimensional Lebesgue measure on the light cone

$$\{x \in \mathbb{R}^4 | \ x_4^2 = x_1^2 + x_2^2 + x_3^2\}.$$