

# An abstract approach to the Crouzeix conjecture

Kamil Gacek

Talk given at Seminar on Complex Analysis,  
meeting 2320, March 7th, 2022

In 2004 Michael Crouzeix conjectured for all complex polynomials and  $n \times n$  matrix  $T$  the following inequality holds:  $\|p(T)\| \leq 2 \max |p(z)|$ , where  $z$  belongs to numerical range of  $T$  ( $W(T)$ )—in other words,  $W(T)$  is a 2-spectral set for  $T$ . In the seminar we will propose an approach to this conjecture through the study of certain continuous (unital) homomorphisms  $\Theta : A \rightarrow M_n(\mathbb{C})$ , where  $A$  is a uniform algebra and  $M_n(\mathbb{C})$  is the algebra of complex  $n \times n$  matrices. Additionally, suppose that there exists antilinear contraction  $\alpha : A \rightarrow A$  such that  $\|\Theta(f) + \Theta(\alpha(f))^*\| \leq 2\|f\|$  for  $f \in A$ . We conjectured that under extra assumption  $\alpha(1) = 1$  we may conclude that  $\|\Theta\| \leq 2$  what would yield a positive solution to the Crouzeix conjecture.