Docquier-Grauert tubular neighbourhood theorem for admissible pairs (based on paper by Forstnerič)

Arkadiusz Lewandowski

We shall consider *admissible* subsets $S \subset X$ of complex manifolds.

Definition. A subset $S = K \cup M \subset X$ is *admissible* if:

(1) K is compact and has a Stein neighbourhood $U \subset X$ such that K is $\mathcal{O}(U)$-convex,
(2) M is embedded Stein submanifold of X,
(3) $K \cap M$ is compact and $\mathcal{O}(M)$-convex.

A pair (K, M) is then called *admissible*.

We discuss the following

Theorem (Forstnerič, 2022). *Let $S = K \cup M \subset X, S' = K' \cup M' \subset X'$ be admissible sets in complex manifolds of the same dimension and let $F : S \to S'$ be a homeomorphism such that $F|_M : M \to F(M) = M'$ is biholomorphic, and with the property that F extends to a biholomorphism from a neighbourhood of K onto a neighbourhood of K'. Assume that there is a topological isomorphism $\Theta : \nu_{M,X} \to \nu_{M',X'}$ of the normal bundles, which is given over a neighbourhood of $K \cap M$ by the differential of F. Then, for any $\varepsilon > 0$ there are an open Stein neighbourhood $\Omega \subset X$ of S and a biholomorphic map $\Phi : \Omega \to \Phi(\Omega) \subset X'$ such that $\Phi|_M = F|_M$ and* $\sup_{x \in K} \mathrm{dist}_{X'}(\Phi(x), F(x)) < \varepsilon$

We shall present its applications, and the main ideas and tools of its proof.