A bounded domain $U \subset \mathbb{C}^n$ is said to satisfy the $\Gamma(C, \alpha, r)$ property for some $C, r > 0, \alpha \in (0, 1)$ if the following happens:

(1) For any point $z \in U$ close to ∂U there is a point $p \in \partial U$, such that after an affine change of coordinates p is identified with 0 and z lies on the positive axis $\{(0, \cdots, 0, t) | \; t > 0\}$;

(2) the set
$$\Gamma(C, \alpha, r) := \{z \mid Re(z_n) > C(Im^2(z_n) + ||z'||^2)^{\alpha/2}, ||z|| < r\}$$
is contained in U;

(3) $||z - p|| < r/2$.

Rouglhy speaking the condition means that at every boundary point U admits an inscribed cusp of fixed opening and size. It can be shown that any bounded domain with Hölder continuous boundary is a $\Gamma(C, \alpha, r)$-domain for some parameters C, α and r.

The main result is the following decay estimate for negative plurisubharmonic functions ρ defined on a $\Gamma(C, \alpha, r)$-domain:

If $\rho \in PSH^-(U)$ then near ∂U one has
$$\rho(z) \leq -Bexp(-\frac{A}{dist_{\partial U}(z)^{1/\alpha - 1}}),$$
where $B > 0$ is some constant, while $A = \frac{\pi C^{1/\alpha}}{2(1/\alpha - 1)}$.

Such an extimate resembles the decay estimate near the boundary provided by the classical Hopf lemma. The novelty here is that such cusps $\Gamma(C, \alpha, r)$ are thin at the origin whenever $n \geq 2$ and hence classical potential theoretic barrier arguments break down. Instead a reduction to the one dimensional case is being utilized and a careful estimate of the one dimensional Green function is used.