On the radial dilation of outer functions in Dirichlet-type spaces in the unit disk of $\mathbb{C}$.

Abstract. Considering Dirichlet-type spaces $(D_\alpha(\mathbb{D}), ||\cdot||_\alpha)_{\alpha\in\mathbb{R}}$ one can make conclusions for the usual Bergman, Hardy and Dirichlet space. We study cyclic vectors with respect to the shift operator. One can identify cyclic vectors using the radial dilation method. The radial dilation of f is given by f/f_r, where $f_r(z) = f(rz)$, $r \in (0, 1)$. The Brown-Shields conjecture claims that a function in the Dirichlet space is cyclic if and only if it is outer and the logarithmic capacity of the zero set of its radial limit function is zero. Working with polynomials we get that a polynomial p is cyclic if and only if $p/p_r \to 1$ weakly. This equivalent definition can not be applied on a general function. We compute the Dirichlet-type norm of the radial dilation of an outer function f in terms of the derivatives $(\log|f|^2)^{(j)}(0)$. One hence can play with these terms identifying cyclic functions. On the other hand tools coming out in the arguments can be applied on the radial dilation of polynomials in more than one variable, i.e. in Dirichlet-type spaces in the unit ball of arbitrary dimension.