The talk was devoted to the proof of the following theorem due to Forstneric:

There exists a complex manifold M that admits a strictly plurisubharmonic function, but no nonconstant holomorphic functions.

The example is constructed as follows: first a projective line V in the projective space $\mathbb{P}^2$ is taken and then it is slightly deformed (in a smooth real non-holomoprhic way) to a real 2-dimensional surface S with only isolated complex points. It was shown that a sufficiently small nieghborhood of S carries a strictly plurisubharmonic function which is essentially the squared distance function to S (with respect to any metric) suitably modified near the complex points.

The proof that such a neighborhood does not admit nonconstant holomorphic functions is much more complicated. First of all Fujita's theorem implies that the envelope of holomorphy os such a domain is Stein. Then a relatively compact neighborhood of S in the envelope of holomorphy can be embedded in a complex Kähler surface X with $b_2^+(X) > 1$. This is turn implies, through Seiberg-Witten theory, that the genus $g(S)$ has to satisfy the inequality

$$g(S) - 2 \geq [S]^2 + |K_X.S|$$

the latter products being computed in homology. Easy computation finally implies that $g(S) \geq 3$, while S is a small deformation of a sphere, and hence $g(S) = 0$, contradiction.