Sendov's conjecture

Introduction to the problem and related results

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This talk, spanning over two meetings, is the first one in the joint effort to understand the paper of Terence Tao on the following conjecture of Sendov:

Let

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = a_n \prod_{j=1}^{n} (z - z_j)$$

be a complex polynomial ($P \in \mathcal{P}$) of degree $n \geq 2$ with all its zeros in the closed unit disc $\overline{\mathbb{D}}$ (i.e., $z_j \in \overline{\mathbb{D}}$, $j = 1, \ldots, n$).

Conjecture of Sendov'1958: For each $j \in \{1, \ldots, n\}$, the closed disc $\overline{\mathbb{D}}(z_j; 1)$ contains a critical point of P.

We present

1. The history of the problem.
2. General theorems for the locus of zeros of polynomials.
3. Relations with other conjectures.
4. Partial results on Sendov's conjecture and what was known before the result of Tao.

In particular I will show that:

- The conjecture holds if one takes slightly bigger discs (with radii $1.07538285\ldots$ instead of 1).
- The conjecture holds if one of the roots is at zero.
- The conjecture holds if the degree of the polynomial does not exceed 5.

Also, I introduce some technical facts to be utilized later in the talks directly discussing the paper of Tao.

I will mainly follow the book of Rahman and Schmeisser: *Analytic Theory of Polynomials*, London Mathematical Society Monographs, London, **26**, 2002