A new projection operator onto L^p Bergman spaces of Reinhardt domains

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The well-known Bergman projection of a domain $\Omega \subset \mathbb{C}^n$ is the orthogonal projection from $L^2(\Omega)$ onto its holomorphic subspace, which we call the Bergman space and denote by $A^2(\Omega)$. The Bergman projection and its integral kernel (the Bergman kernel) can be useful tools in the study of other holomorphic function spaces on Ω, particularly when the domain satisfies nice regularity conditions (e.g. pseudoconvex with smooth boundary). But the presence of boundary singularities on Ω can force the mapping behavior of the Bergman projection in other function spaces to badly deteriorate, greatly limiting its utility on non-smooth domains.

This talk is concerned with the problem of understanding L^p-Bergman spaces, $p \neq 2$, on domains with boundary singularities (that is, spaces of holomorphic L^p functions, which we denote by A^p). Let $1 < p < \infty$, and $\Omega \subset \mathbb{C}^n$ any pseudoconvex Reinhardt domain (with no assumptions on boundary smoothness). Using the metric geometry of $A^p(\Omega)$ we construct a new integral kernel operator generalizing the Bergman projection. On an interesting class of non-smooth pseudoconvex domains where the Bergman projection has serious deficiencies, we prove that this new operator has much better mapping regularity and can thus be used as a substitute tool with which to study A^p spaces. Applications to holomorphic duality theorems and the ability to define a new metric generalizing the Bergman metric will also be discussed.