

Gauss-Lucas theorem in dynamics of complex polynomials

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The classical Gauss-Lucas theorem says that for a (nonconstant) polynomial p with complex coefficients all zeros of the derivative p' lie in the convex hull of the zeros of p . We will start this talk by presenting two modern variants of this theorem, due respectively to L. Hörmander and W. P. Thurston. Using these variants, we will prove that for every complex polynomial p of degree $d \geq 2$ the convex hull H_p of the Julia set J_p of p satisfies $p^{-1}(H_p) \subset H_p$. This settles positively a recent conjecture by Per Alexandersson. We will also characterize polynomials for which the equality $p^{-1}(H_p) = H_p$ is achieved. We will further discuss some difficulties in generalizing such results to non-polynomial rational functions. Necessary background in complex dynamics will be provided.