On the boundary zeros of a zero-free polynomial in $\mathbb{B}_n$

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Abstract: We study the cyclicity problem of polynomials in Dirichlet-type spaces in the unit ball of $\mathbb{C}^n$ with respect to the shift operators. The cyclicity of a function is intimately connected with its zero set restricted on the unit sphere. The boundary zeros of a zero-free polynomial in $\mathbb{B}_n$ are those zeros lying in the unit sphere. The Bruhat–Cartan–Wallace Curve Selecting Lemma was crucial to identify non-cyclicity in the two dimensional case. We used it as follows: if a polynomial has infinitely many boundary zeros then there is an analytic curve contained in them. The analytic curve is of real dimension 1 which it is not convenient whenever $n > 2$. We go further giving an analogue description of the boundary zeros in arbitrary dimension.