In the first part we discussed the theory of the classical distance function

$$d_K(x) := \{||x - y|| \mid y \in K\}$$

to a closed set K in an Euclidean space $\mathbb{R}^m$.

In particular we explained the case when $K = \partial\Omega$ is the boundary of a domain. The *medial axis* Γ defined as the set of points from which closest point to $\partial\Omega$ is not unique is shown to be the set where $d_{\partial\Omega}$ fails to be differentiable. By an old result of Erdös Γ is always $(m-1)$ rectifiable.

It's Euclidean closure Σ may be much larger. We described an example of Mantegazza and Mennucci that it can have positive Lebesgue measure even for domains with $C^{1,1}$ boundary. A theorem of Crasta and Malusa shows hovewer that for domains with at least C^2 boundary Σ is always Lebesgue null set.

In the second part we discussed a direct proof of the Oka lemma - that for holomorphically convex domains the function $-log(d_{\partial\Omega})$ is plurisubharmonic. We explained the role of the singular set Σ in a direct proof of this lemma.