Stahl's theorem (solving a conjecture by Bessis-Moussa-Villani) can be stated in several equivalent forms. Some of them include:

Theorem: Let A and B be positive definite Hermitian matrices. Then for every $m \in \mathbb{N}$ the polynomial

$$t \longmapsto tr[(A + tB)^m]$$

has nonnegative coefficients

Theorem: Leta A, B be Hermitian matrices of dimension n. Assume $B \geq 0$. Then the function

$$\mathbb{R} \ni t \longmapsto tr(e^{A-tB})$$

is completely monotone i.e. can be written as

$$\int_0^\infty e^{-st} d\mu_{A,B}(s)$$

for some positive measure $\mu_{A,B}$.

While special cases of the above results were known, the full proof appeared only in Stahl's paper.

In the talk a sketch of the proof was presented. It made crucial use of the equation

$$g(\lambda, t) = det(\lambda I_n - (A - tB)) = 0$$

for $t \in \mathbb{C}$. If g is irreducible as a polynomial of 2 variables (if not we take an irreducible factor if it) the solution $\lambda(t)$ is a multivalued holomorphic function. Using this and basic complex analysis the existence of a (non-signed) measure $\mu_{A,B}$ follows easily. Then a subtle analysis made on the Riemann domain associated to λ justifies the nonnegativity of $\mu_{A,B}$.