Holomorphic automorphisms of Markov surfaces

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The Diophantine solutions of the equation $x^2 + y^2 + z^2 = 3xyz$, were originally considered by Markov and are now called Markov triples. Later, this equation was studied over the complex numbers by algebraic geometers; The group of algebraic automorphisms is discrete and acts transitively on the Markov triples. We describe the identity component of the group of volume-preserving holomorphic automorphisms. In contrast to the algebraic case, this group is infinite-dimensional and interpolates any permutation of (ordered) Markov triples. The results can be extended to so-called Markov surfaces of the form $x^2 + y^2 + z^2 - 3xyz - Ax - By - Cz - D = 0$.