Membership of Rational Inner Functions in Dirichlet spaces on the bidisc.

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Abstract:

Let $p \in \mathbb{C}[z_1, z_2]$ a polynomial which has no zeros on the bidisc $\mathbb{D}^2$ but vanishes on at least one point on the bitorus $\mathbb{T}^2$. Then, consider its reflection $\tilde{p}$ and define $\varphi = \frac{\tilde{p}}{p}$. These functions are Rational and also have the property of being Inner, in the sense that $|\varphi| = 1$ on $\mathbb{T}^2$ almost everywhere in the Lebesgue sense. The main topic of today's talk is to study the membership of these function on sever Dirichlet-type spaces on the bidisc. The spaces that we will consider in this talk are the following:

$$\mathfrak{D}(\mathbb{D}^2) := \left\{ f \in \mathcal{O}(\mathbb{D}^2) : \int_{\mathbb{D}^2} \left| \frac{\partial(z_1 z_2 f(z_1, z_2))}{\partial z_1 \partial z_2} \right|^2 dA(z_1) dA(z_2) < \infty \right\}$$

and

$$\mathcal{D}_{n,m}(\mathbb{D}^2) = \left\{ f \in \mathcal{O}(\mathbb{D}^2) : \sup_{r<1} \int_{\mathbb{T}} \int_{\mathbb{D}} \left| \frac{\partial^n f}{\partial z_1^n}(z_1, re^{i\theta}) \right|^2 dA(z_1) d\theta + \sup_{r<1} \int_{\mathbb{T}} \int_{\mathbb{D}} \left| \frac{\partial^m f}{\partial^m z_2}(re^{i\theta}, z_2) \right|^2 dA(z_2) d\theta < +\infty \right\}$$

Some related references are shown below

References

[1] K.Bickel, J.E. Pascoe, A.Sola, *Derivatives of Rational Inner Functions and integrability at the boundary,* Proceedings of the London Mathematical Society, Vol. 116, Issue 2, pp.281-329

[2] G.Knese, *Rational Inner Functions in the Schur-Agler class of the polydisc*, Publicacions Matemàtiques, Vol. 55, No. 2 (2011), pp. 343-357