Harmonic Bergman spaces and subharmonic L^p functions
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This talk spans over two meetings (9.03.2026, 16.03.2026).

In the first part of the talk, I introduce various notions of small sets in terms of (generalized) Hausdorff measures and (generalized) capacities. After that, I survey what is known about the characterization of planar domains allowing non-trivial L^p holomorphic functions (full characterization exists), spatial holomorphic functions (only partial characterization is available), planar and spatial harmonic functions (again, full characterization exists). After that, I present my own result which contains a full characterization of planar and spatial domains, allowing non-trivial subharmonic functions in L^p. The result reads:

Proposition 1.1. *For $\Omega \subseteq \mathbb{C} \cong \mathbb{R}^2$, we have:*

- *If the complement $\mathbb{C} \setminus \Omega$ is non-polar then the class $SH(\Omega) \cap L^\infty(\Omega)$ is non-trivial*
- *If the complement $\mathbb{C} \setminus \Omega$ is polar then $SH(\Omega) \cap L^\infty(\Omega)$ contains only constants*
- *If the complement $\mathbb{C} \setminus \Omega$ contains at least one point then the class $SH(\Omega) \cap L^p(\Omega), \quad 1 \leq p < \infty$ is non-trivial*
- *If $\Omega = \mathbb{C}$ then $SH(\Omega) \cap L^p(\Omega), \quad 1 \leq p < \infty$ contains only the zero function*

A proof in the planar case is presented.

In the second part, I prove the spatial case which reads:

Proposition 1.2. *For $\Omega \subseteq \mathbb{R}^n$, $n \geq 3$, we have:*

- *If Ω is any domain and $p \in \left(\frac{n}{n-2}, \infty\right) \cup \{\infty\}$ then the class $SH(\Omega) \cap L^p(\Omega)$ is non-trivial*
- *If the complement $\mathbb{R}^n \setminus \Omega$ contains at least one point and $p \in \left[1, \frac{n}{n-2}\right)$ then the class $SH(\Omega) \cap L^p(\Omega)$ is non-trivial*
- *If $\Omega = \mathbb{R}^n$ and $p \in \left[1, \frac{n}{n-2}\right]$ then $SH(\Omega) \cap L^p(\Omega)$ contains only the zero function*
- *If $p = \frac{n}{n-2}$ and $C_{\alpha,q}(\mathbb{R}^n \setminus \Omega) = 0$, that is, the complement $\mathbb{R}^n \setminus \Omega$ has vanishing (α, q)- capacity, where $\alpha = 2$ and $q = \frac{n}{2}$, then $SH(\Omega) \cap L^p(\Omega)$ contains only the zero function*

- *If* $p = \frac{n}{n-2}$ *and* $C_{\alpha,q}(\mathbb{R}^n \setminus \Omega) > 0$, *that is, the complement* $\mathbb{R}^n \setminus \Omega$ *has positive* (α, q)*- capacity, where* $\alpha = 2$ *and* $q = \frac{n}{2}$, *then* $SH(\Omega) \cap L^p(\Omega)$ *is non-trivial*

An almost complete characterization of planar and spatial domains allowing non-trivial subharmonic functions in the Sobolev space $W^{1,p}$ is also presented. At the end of the talk, I briefly discuss why such a characterization for plurisubharmonic functions on domains $\Omega \subseteq \mathbb{C}^n$ is probably impossible, except for the trivial cases, e.g., when the domain Ω is planar.