Identity principles in several variables
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This talk spans over two meetings (16.03.2026, 23.03.2026).

In the first part of the talk, I recall the classical identity principle for holomorphic functions of one variable, and the relations between non-uniqueness sets for holomorphic functions and zero sets for holomorphic functions. It turns out that there is no non-trivial description of the corresponding non-uniqueness and zero sets for harmonic functions. After that, I survey what is known about the uniqueness and non-uniqueness sets for holomorphic functions of several variables (full characterization exists, by an old theorem of Viola, but it is not topological, unlike the situation in dimension 1). After that, I formulate my own result which contains a full characterization of the non-uniqueness sets of real-analytic subharmonic, real-analytic plurisubharmonic, and real-analytic locally convex functions. It turns out that these are the same as for general real-analytic functions. More specifically, the result reads:

Proposition 1.1.

- *If $\Omega \subseteq \mathbb{R}^n$ is an arbitrary domain then the uniqueness sets of $SH(\Omega) \cap C^\omega(\Omega)$ and $C^\omega(\Omega)$ are the same, and are exactly the sets which are not contained in a $\mathbb{C}$-analytic subset of Ω.*

- *If $\Omega \subseteq \mathbb{C}^m$ is an arbitrary domain then the uniqueness sets of $PSH(\Omega) \cap C^\omega(\Omega)$ are again exactly the sets which are not contained in a $\mathbb{C}$-analytic subset of Ω, treated as a real domain in $\mathbb{R}^{2m}$.*

- *If $\Omega \subseteq \mathbb{C}^m$ is a pseudoconvex domain and $X \subseteq \Omega$ is contained in a $\mathbb{C}$-analytic subset of Ω then one can find two distinct functions $\psi_1, \psi_2 \in PSH(\Omega) \cap C^\omega(\Omega)$ that coincide on X, which are moreover both exhaustion functions of Ω.*

- *If $\Omega \subseteq \mathbb{R}^n$ is an arbitrary domain then the uniqueness sets of $CVX_{loc}(\Omega) \cap C^\omega(\Omega)$ are again exactly the sets which are not contained in a $\mathbb{C}$-analytic subset of Ω.*

- *If $\Omega \subseteq \mathbb{C}^m$ is a convex domain and $X \subseteq \Omega$ is contained in a $\mathbb{C}$-analytic subset of Ω then one can find two distinct functions $\psi_1, \psi_2 \in CVX(\Omega) \cap C^\omega(\Omega)$ that coincide on X, which are moreover both exhaustion functions of Ω.*

Near the end of the first part, I introduce the necessary notions from real-analytic geometry, that is, real-analytic sets and $\mathbb{C}$-analytic sets, discuss the differences between them, and provide some examples.

In the second part, I prove the stated proposition, and discuss some corollaries.