Approximation of analytic sets with proper projection by algebraic sets

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Let Z, Z_ν, for $\nu \in \mathbf{N}$, be purely k-dimensional analytic subsets of an open subset of $\mathbf{C}^m$. We say that the sequence $\{Z_\nu\}$ converges to Z in the sense of holomorphic chains if the sequence of currents of integration over Z_ν converges to the current of integration over Z in the weak-* topology. The aim of the talk is to prove the following

Theorem 0.1 *Let U be a Runge domain in $\mathbf{C}^k$ and let X be an analytic subset of $U \times \mathbf{C}^n$ of pure dimension k with proper projection onto U. Then there is a sequence $\{X_\nu\}$ of algebraic subsets of $\mathbf{C}^k \times \mathbf{C}^n$ of pure dimension k such that $\{X_\nu \cap (U \times \mathbf{C}^n)\}$ converges to X in the sense of holomorphic chains.*

The proof of Theorem 0.1 is based on two results. Firstly, it is known that every purely dimensional analytic set with proper and surjective projection onto a Runge domain can be approximated by Nash sets. Secondly, we show that every complex Nash set with proper projection onto a Runge domain can be approximated by algebraic sets as stated in the following

Proposition 0.2 *Let Y be a Nash subset of $\Omega \times \mathbf{C}$ of pure dimension $k < m$, with proper projection onto Ω, where Ω is a Runge domain in $\mathbf{C}^{m-1}$. Then there is a sequence $\{Y_\nu\}$ of algebraic subsets of $\mathbf{C}^{m-1} \times \mathbf{C}$ of pure dimension k such that $\{Y_\nu \cap (\Omega \times \mathbf{C})\}$ converges to Y in the sense of holomorphic chains.*