

Seminar on Geometric Function Theory

Meeting 29, 19th October 2009

lecture: **Some open problems in Geometric Function Theory**

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- (1) Is a bounded smooth pseudoconvex domain  $D \subset \mathbb{C}^n$   $k_D$ -complete or  $c_D$ -complete?
- (2) Let  $D \subset \mathbb{C}^n$  be a bounded smooth pseudoconvex domain.
  - (a) Does for any  $\varepsilon > 0$  and  $z_0 \in \partial D$  exist a constant  $c > 0$  such that  $\kappa_D(z; n(z_0)) \geq \frac{c}{(d_D(z))^{1-\varepsilon}}$  for any  $z \in (z_0 - n(z_0)) \cap D$ , where  $n(z_0)$  is a normal vector at  $z_0$ ?
  - (b) Does for  $n = 2$  and  $z_0 \in \partial D$  exist a constant  $c > 0$  such that  $\kappa_D(z; n(z_0)) \geq \frac{c}{d_D(z)}$  for any  $z \in (z_0 - n(z_0)) \cap D$ , where  $n(z_0)$  is a normal vector at  $z_0$ ?
- (3) Is any bounded pseudoconvex balanced domain  $D \subset \mathbb{C}^2$  with a continuous Minkowski functional  $k_D$ -complete?
- (4) Is any bounded pseudoconvex balanced domain  $D \subset \mathbb{C}^n$  with a smooth Minkowski functional  $k_D$ -complete?
- (5) Is any bounded domain with continuous boundary hyperconvex?
- (6) Is any pseudoconvex Hartogs domain  $D$  with  $b_G$ -complete basis  $G$   $b_D$ -complete?
- (7) Is the Lempert theorem valid for  $\mathbb{C}$ -convex domains?
- (8) Can any  $\mathbb{C}$ -convex domain be exhausted by smooth  $\mathbb{C}$ -convex domains?
- (9) Can  $\mathbb{G}_2$  be exhausted by smooth  $\mathbb{C}$ -convex domains?
- (10) Is the Lempert theorem valid for the tetrablock?
- (11) Is the Green function continuous for pseudoconvex Reinhardt domains?
- (12) Can  $\{(z, w) \in \mathbb{C}^2 : (\operatorname{Re} z^2)^2 + |z|^2 + |w|^2 < 1\}$  be exhausted by domains biholomorphic to convex ones?
- (13) Does for any pseudoconvex domain exist  $N$  such that the infimum in the definition of the Kobayashi pseudodistance for  $D$  is taken only over  $N$  discs?
- (14) Is the infimum in the Poletsky's definition of the Green function of hyperconvex domains always attained?
- (15) Does the Green function of a hyperconvex domain converge to zero as the pole tends to the boundary?
- (16) Is the Green function of a strictly pseudoconvex domain with smooth boundary of class  $\mathcal{C}^2$ ?