BEHAVIOR OF THE KOBAYASHI DISTANCE NEAR A BOUNDARY OF PSEUDOCONVEX REINHARDT DOMAINS

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The following theorems connecting the Kobayashi (pseudo)distance k_D and a distance to the boundary d_D are proved.

THEOREM 1. *Let $D \subset \mathbb{C}^n$ be a pseudoconvex Reinhardt domain. Fix $z_0 \in D$ and $\zeta_0 \in \partial D$. Then for some constant C the inequality*

$$k_D(z_0, z) \leq -\log d_D(z) + C$$

holds if $z \in D$ tends to ζ_0. Additionally, for $\zeta_0 \in \mathbb{C}_*^n$ the estimate can be improved to

$$k_D(z_0, z) \leq -\frac{1}{2} \log d_D(z) + C'$$

where C' is a constant.

THEOREM 2. *Let $D \subset \mathbb{C}^n$ be a pseudoconvex Reinhardt domain. Fix $z_0 \in D$ and $\zeta_0 \in \partial D \cap \mathbb{C}_*^n$. Then for some constant C the inequality*

$$k_D(z_0, z) \geq -\frac{1}{2} \log d_D(z) + C$$

holds if $z \in D$ tends to ζ_0.

THEOREM 3. *Let $D \subset \mathbb{C}^n$ be a $\mathcal{C}^1$-smooth pseudoconvex Reinhardt domain. Fix $z_0 \in D$ and $\zeta_0 \in \partial D \setminus \mathbb{C}_*^n$. Then for some constant C the inequality*

$$k_D(z_0, z) \geq -\frac{1}{2} \log d_D(z) + C$$

holds if $z \in D$ tends non-tangentially to ζ_0.

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