A Remark on Gromov's method and Bennequin's problem

Denote $D_R^+ := \{z \in \mathbb{C} : |z| < R, \operatorname{Re} z \geq 0\}$.

Twierdzenie 1. *Let $L \subset \mathbb{C}^N$ be a totally real compact m-dimensional $\mathcal{C}^\infty$ submanifold. Let $R > 0$, $r \in (1, \infty) \setminus \mathbb{N}$, $(f_k)_{k=1}^\infty \subset \mathcal{C}^{r+1}(D_R^+, \mathbb{C}^N)$ be such that*

- $f_k(D_R^+ \cap \mathbb{R}) \subset L$,
- f_k *tends uniformly on* D_R^+ *to some* $f : D_R^+ \to \mathbb{C}^N$,
- $g_k := \frac{\partial f_k}{\partial \overline{\lambda}}$ *tends to some* $g : D_R^+ \to \mathbb{C}^N$ *in the space* $\mathcal{C}^r(D_R^+, \mathbb{C}^N)$

Then $f \in \mathcal{C}^{r+1}(D_R^+, \mathbb{C}^N)$, $g = \frac{\partial f}{\partial \overline{\lambda}} \in \mathcal{C}^r(D_R^+, \mathbb{C}^N)$, and for every $\rho \in (0, R)$ there is $f_k \to f$ in the space $\mathcal{C}^{r+1}(\overline{D_\rho^+}, \mathbb{C}^N)$.