On the Boundary Regularity of Analytic Discs

Evgeni M. Chirka ; Bernard Coupet ; Alexandre B. Sukhov

In this talk we will study the boundary behavior of analytic discs near the zero set of a nonnegative plurisubharmonic function or a totally real submanifold of $\mathbb{C}^n$. We denote the cluster by

$$C(f;a) = \{p \in \overline{\Omega} \text{ such that } \exists\ (a_n)_n \subset \Delta \text{ with } a_n \to a \text{ and } f(a_n) \to p\}$$

Our main result is the following.

Theorem 1. *Let Ω be a complex manifold ; ρ a plurisubharmonic function in Ω and $f : \Delta \to \Omega$ a holomorphic map of the unit disc $\Delta \subset \mathbb{C}$ into Ω such as $\rho o f \geqslant 0$ and $\rho o f(\zeta) \to 0$ as $\zeta \in \Delta$ tends to an open arc $\gamma \subset \partial\Delta$. Assume that for a certain point $a \in \gamma$ the cluster set $C(f;a)$ contains a point $p \in \Omega$ such that ρ is strictly plurisubharmonic in a neighborhood of p. Then f extends to a Holder $\frac{1}{2}-$continuous mapping in a neighborhood of a on $\Delta \cup \gamma$. If moreover $\rho \geq 0$ and the function ρ^θ is plurisubharmonic in a neiborhood of p for some $\theta \in [\frac{1}{2}; 1]$; then f is Holder $\frac{1}{2\theta}-$continuous (Lipschitz ; if $\theta = \frac{1}{2}$) in a neighborhood of a on $\Delta \cup \gamma$.*

In conclusion we 'll indicate as an application of this theorem that the area of the analytic disc $f(\Delta)$ attached to a C^1 smooth totally real manifold is finite

Références

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