An "annulus" formula for the relative extremal function

Arkadiusz Lewandowski

Abstrakt. The aim of this talk is to present the following "annulus" formula for the relative extremal function

Theorem 0.1. *Let $D \subset\subset X$, where for the couple (D, X) at least one of the following two conditions is satisfied:*

(A) *D is an irreducible, locally irreducible weakly parabolic Stein space with some potential g and X is a Stein space,*

(B) *D is a Stein manifold and X is a Josefson manifold.*

Let $A \subset D$ be nonpluripolar. Define

$$\Delta(r) := \{z \in D : h^{\star}_{A,D}(z) < r\}, \quad \Delta[r] := \{z \in D : h^{\star}_{A,D}(z) \leq r\}, \quad r \in (0, 1].$$

Then for $0 < r < s \leq 1$ we have

$$h^{\star}_{\Delta(r),\Delta(s)} = \max\left\{0, \frac{h^{\star}_{A,D} - r}{s - r}\right\} \quad \text{on } \Delta(s).$$

The formula given in the result above was known before in the context of Riemann domains of holomorphy over $\mathbb{C}^n$ ([JP2]). We give a generalization of it to the context of Stein manifolds and even Stein spaces, which allows, for example, to conclude the extension theorem for generalized (N, k)-crosses ([L]) in the context of Stein manifolds.

References

[AH] O. Alehyane,J.-M. Hecart, *Propriété de stabilité de la fonction extrémale relative*, Potential Anal. 21 (2004), 363-373.

[AZ] O. Alehyane, A. Zeriahi, *Une nouvelle version du théorème d'extension de Hartogs pour les applications séparément holomorphes entre espaces analytiques*, Ann. Polon.Math. 76 (2001), 245–278.

[B] E. Bedford, *The operator $(dd^c)^n$ on complex spaces*, Séminaire d'Analyse Lelong-Skoda et Colloque Wimereux, Lecture Notes in Math. 919, Springer-Verlag, Berlin 1981.

[BT] E. Bedford, B.A. Taylor, *A new capacity for plurisubharmonic functions* Acta Math., 149 (1982), 1-40.

[D] J.P. Demailly, *Mesures de Monge-Ampère et caractérisation géométrique des variétés algébriques affines*, Prépublication de l'Institut Fourrier 22, 1985.

[EP] A. Edigarian, E.A. Poletsky, *Product property of the relative extremal function*, Bull. Sci. Acad. Polon. 45 (1997), 331-335.

[F] J.E. Fornaess, *An increasing sequence of Stein manifolds whose limit is not Stein*, Math. Ann. 223 (1976), 275-277.

[FN] J.E. Fornaess, R. Narasimhan, *The Levi problem on complex spaces with singularities*, Math. Ann. 248 (1980), 47-72.

[GR] R.C. Gunning, H. Rossi, *Analytic functions of several complex variables*, Prentice-Hall, Englewood Cloffs, NJ, 1965.

[JP1] M. Jarnicki, P. Pflug, *An extension theorem for separately holomorphic functions with pluripolar singularities*, Trans. Amer. Math. Soc. 355 (2003), 1251–1267.

[JP2] M. Jarnicki, P. Pflug, *A new cross theorem for separately holomorphic functions*, Proc. Amer. Math. Soc. 138 (2010), 3923–3932.

[JP3] M. Jarnicki, P. Pflug, *Separately analytic functions*, EMS Publishing House, 2011.

[K] M. Klimek, *Pluripotential theory*, London Math. Soc. Monogr. (N.S.) 6, Oxford University Press, New York, 1991.

[L] A. Lewandowski, *An extension theorem with analytic singularities for generalized (N, k)-crosses*, Ann. Polon. Math. 103 (2012), 193-208.

[Lo] S. Lojasiewicz, *Introduction to complex analytic geometry*, Translated from the Polish by Maciej Klimek, Birkhäuser Verlag, 1991.

[N1] R. Narasimhan, *The Levi problem for complex spaces*, Math. Ann. 142 (1961), 355-365.

[N2] R. Narasimhan, *The Levi problem for complex spaces II*, Math. Ann. 146 (1962), 195-216.

[S] A. Sadullaev, *Plurisubharmonic measures and capacities on complex manifolds*, Russian Math. Surveys 36 (1981), 61-119.

[Sm] P.A.N. Smith, *Smoothing plurisubharmonic functions on complex spaces*, Math. Ann. 273 (1986), 397-413.

[St] W. Stoll, *The characterization of strictly parabolic spaces*, Compositio Math. 44, no 1-3 (1981), 305-373.

[Z1] A. Zeriahi, *Fonctions plurisousharmoniques extrémales. Approximation et croissance des fonctions holomorphes sur des ensembles algébriques* Thèse de Doctorat d'État, Sciences, U.P.S. Toulouse, 1986.

[Z2] A. Zeriahi, *Fonction de Green pluricomplexe à pôle à l'infini sur un espace de Stein parabolique et applications*, Math. Scand. 69 (1991), 89-126.