0.1 Proper holomorphic self-maps of plane regions

The following theorems due to C. Mueller and W. Rudin will be proved.

Theorem 1. *Suppose* $3 \leq p < \infty$, Ω *is a region in* $\mathbb{C}$*, and* $K_1, ..., K_p$ *are the components of* $\partial\Omega$*.Then* $PRH(\Omega) = Aut(\Omega)$*, and* $Aut(\Omega)$ *is a finite group.*

Theorem 2. *For a rational function* $f(z) = cz - \sum_{i=1}^{m-1} \frac{1}{z-a_i}$ *where* $m \geq 2$*,* c *and* $a_1, ..., a_n$ *are real numbers,* $c > 1$*,* $a_1 < ... < a_{m-1}$ *we define some f-orbits:*
$K_0 = \{x : p_1 < x < p_m\}$ $K_{n+1} = f^{-1}(K_n)$ *for* $n = 0, 1, 2, ...$ *and*
$K = \bigcap_{n=0}^{\infty} K_n$ *where* $p_1, ..., p_m$ *are fixed point of* f *in* $\mathbb{C}$
$A_0 = \{a_1, ..., a_m\}$ $A_{n+1} = f^{-1}(A_n)$ *for* $n = 0, 1, 2, ...$ *and*
$A = \bigcap_{n=1}^{\infty} A_n$*. Then*
(i) K *is a Cantor set.*
(ii) The backward f-orbit of any $x \in K_0$ *has* K *as its set of limit points.*
(iii) K *is the only nonempty compact subset of* $\mathbb{R}$ *that is a complete f-orbit.*
(iv) $A \cup \{\infty\}$ *is a minimal complete f-orbit.*
(v) $K \cup A$ *is compact, and* $K \cup A \cup \{\infty\}$ *is a complete f-orbit. Conversely,if* H *is a compact subset of* $\mathbb{R}$ *and* $H \cup \{\infty\}$ *is a complete f-orbit, then* $H = K \cup A$*.*
(vi) If $z_0 \in \Pi$ *then complete f-orbit generated by* z_0 *is a discrete set* $E \subset \Pi$ *which has* $K \cup A \cup \{\infty\}$ *as its set of limit points.*

Theorem 3. *There exist a region* Ω*, whose boundary consist of* $\mathbb{R} \cup \{\infty\}$ *plus countable set of Jordan curves and* $f \in PRH(\Omega)$ *with multiplicity m, where m is preassigned.*