$\mathscr{A}$-crosses

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We shall define a new type of cross-like objects and prove for them the following extension results:

THEOREM 0.0.1. *Let D_j be a Riemann domain of holomorphy over $\mathbb{C}^{n_j}$ and let $A_j \subset D_j$ be locally pluriregular, $j = 1, \ldots, N$. Let $\mathbf{Q} := \mathbb{Q}(\alpha_j^i)((A_j, D_j)_{j=1}^N)$ be an $\mathscr{A}$-cross. Assume that an N-fold cross $\mathbb{X}_{N,1}((A_j, D_j)_{j=1}^N)$ is contained in $\mathbf{Q}$. Then there exist an $m_0 \in \{1, \ldots, N\}$, a domain of holomorphy $G \subset D_1 \times \ldots \times D_{m_0-1} \times D_{m_0+1} \times \ldots \times D_N$, a locally pluriregular subset $B \subset G$, and a 2-fold classical cross $\mathbf{X} = \mathbb{X}(A_{m_0}, B; D_{m_0}, G)$ containing $\tau^{-1}(\mathbf{Q})$, where τ is a mapping which sends a point $(z_{m_0}, z_1, \ldots, z_{m_0-1}, z_{m_0+1}, \ldots, z_N) \in D_{m_0} \times D_1 \times \ldots \times D_{m_0-1} \times D_{m_0+1} \times \ldots \times D_N$ to the point $(z_1, \ldots, z_N) \in D_1 \times \ldots \times D_N$, such that for every function $f \in \mathcal{F} := \mathcal{O}_s(\mathbf{Q})$ (separately holomorphic) there exists a unique function $\widehat{f} \in \mathcal{O}(\widehat{\mathbf{X}})$ (here $\widehat{\mathbf{X}}$ is the envelope of holomorphy of $\mathbf{X}$) with $\widehat{f} = f \circ \tau$ on $\tau^{-1}(\mathbf{Q})$, i.e. $\widehat{\mathbf{Q}} := \tau(\widehat{\mathbf{X}})$ is the envelope of holomorphy of $\mathbf{Q}$.*

THEOREM 0.0.2. *Let D_j be a Riemann domain of holomorphy over $\mathbb{C}^{n_j}$ and $A_j \subset D_j$ be locally pluriregular, compact, and holomorphically convex, $j = 1, \ldots, N$. Put $\mathbf{Q} := \mathbb{Q}(\alpha_j^i)((A_j, D_j)_{j=1}^N)$. Assume that $\mathbb{X}_{N,1}((A_j, D_j)_{j=1}^N) \subset \mathbf{Q}$. Denote by $\widehat{\mathbf{Q}}$ the envelope of holomorphy of $\mathbf{Q}$ (cf. Theorem 0.0.1). Define $M := \mathbf{Q} \cap G$, where G is an analytic subset of an open neighborhood U of $\mathbf{Q}$ contained in $\widehat{\mathbf{Q}}$ with $\operatorname{codim} G \geq 1$ and let $\mathcal{F} := \mathcal{O}_s(\mathbf{Q} \setminus M)$. Then there exist an analytic set $\widehat{M} \subset \widehat{\mathbf{Q}}$ and an open neighborhood $U_0 \subset U$ of $\mathbf{Q}$ such that:*

- *$\widehat{M} \cap U_0 \subset G$,*
- *for any $f \in \mathcal{F}$ there exists an $\widehat{f} \in \mathcal{O}(\widehat{\mathbf{Q}} \setminus \widehat{M})$ such that $\widehat{f} = f$ on $\mathbf{Q} \setminus M$,*
- *$\widehat{M}$ is singular with respect to the family $\{\widehat{f} : f \in \mathcal{F}\}$,*
- *$\widehat{f}(\widehat{\mathbf{Q}} \setminus \widehat{M}) \subset f(\mathbf{Q} \setminus M)$ for any $f \in \mathcal{F}$.*

In particular, the extension result holds for the (N, k)-crosses.