Worm domain

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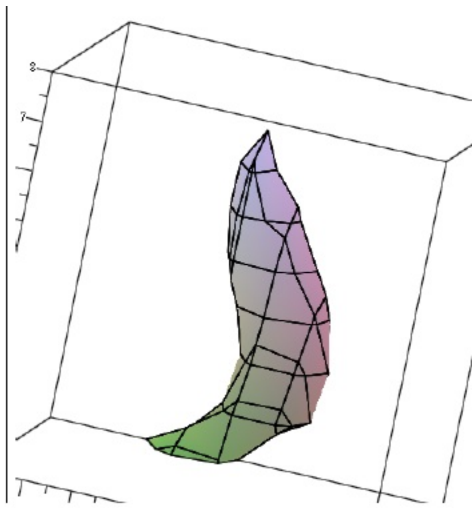
Definition 1 *Suppose that:*

$$\overline{\Omega} = \cap W_j,$$

where

$$W_1 \Supset W_2 \Supset \dots \Supset \dots \overline{\Omega},$$

W_j are smoothly bounded, pseudoconvex domains and Ω is pseudoconvex. A domain having this property is said to have a Stein negihborhood basis.



Definition 2 (Worm domain) *Let $\mathcal{W}$ denote domain:*

$$\mathcal{W} := \{(z_1, z_2) \in \mathbb{C}^2 : |z_1 - \mathrm{e}^{i \log |z_2|^2}|^2 < 1 - \eta(\log |z_2|^2)\}$$

where

1. *$\eta \geq 0$, η is even, η is convex*
2. *$\eta^{-1}(0) = [-\mu, \mu]$*
3. *There exists a number $a > 0$ such that $\eta(x) > 1$ if $|x| > a$.*

4. $\eta'(x) \neq 0$ *if* $\eta(x) = 1$.

Definition 3 *We say* λ *is a bounded plurisubharmonic exhaustion function for a domain* Ω *if:*

- λ *is continuous on* $\overline{\Omega}$
- λ *is strictly plurisubharmonic on* Ω
- $\lambda = 0$ *on* $\partial\Omega$
- $\lambda < 0$ *on* Ω
- *For any* $c < 0$*, the set*

$$\Omega_c := \{z \in \Omega : \lambda(z) < c\} \Subset \Omega$$

Proposition 1 *The domain* $\mathcal{W}$ *is smoothly bounded and pseudoconvex.*

Proposition 2 *The smooth worm domian* $\mathcal{W}$ *has not a Stein neigborhood basis.*

Proposition 3 *There exists no defining funtion* $\tilde{\varrho}$ *for* $\mathcal{W}$ *that is plurisubharmonic on the entire boundary.*

Proposition 4 *Let* $\delta_0 > 0$ *be fixed. Then there exists* $\mu_0 > 0$ *such that, for all* $\mu \geq \mu_0$*, the following holds. If* $\tilde{\varrho}$ *is a defining function for* $W = W_\mu$ *with* $\mu \geq \mu_0$ *and* $\delta > 0$ *is such that* $-(-\tilde{\varrho})^\delta$ *is a bounded plurisubharmonic exhaustion function for* $\mathcal{W}$*, then* $\delta < \delta_0$