Quadratic and robust stability

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= \; & \max_{\triangle \in \mathbf{B}_{\boldsymbol{\Delta}}} \rho \left(\left(\sqrt{P} \mathscr{S}(M, \triangle) \sqrt{P}^{-1} \right)^* \left(\sqrt{P} \mathscr{S}(M, \triangle) \sqrt{P}^{-1} \right) \right) - 1
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= & \max_{\triangle\in\mathbf{B}_{\boldsymbol{\Delta}}} \bar{\sigma}\left(\sqrt{P}\mathscr{S}(M,\triangle)\sqrt{P}^{-1}\right)-1 =: \gamma-1.
\end{aligned}$$

In particular, after coordinate change by $\sqrt{P}$ the map $\mathscr{S}(M, \triangle_k)$ is a contraction and $\|\sqrt{P}x_{k+1}\|_2 \leq \gamma \|\sqrt{P}x_k\|_2$.

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Let
$$G(z) := M_{22} + M_{21}(zI_n - M_{11})^{-1}M_{12}, \quad z \in \mathbb{C}.$$

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$$M^P := \begin{bmatrix} \sqrt{P} M_{11} \sqrt{P}^{-1} & \sqrt{P} M_{12} \\ M_{21} \sqrt{P}^{-1} & M_{22} \end{bmatrix},$$

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Then the following conditions are equivalent.

(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
$\max_{\triangle \in \mathbf{B_\Delta}} \bar{\sigma}(\sqrt{P}\mathscr{S}(M,\triangle)\sqrt{P}^{-1}) < 1.$

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For the general block structure the conditions (1) and (3) are incomparable.

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(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
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(2) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \max_{\delta_2 \in \overline{\mathbb{D}}} \bar{\sigma}(\mathscr{S}(M^P, \delta_2 I_m)) < 1.$

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Let $M \in \mathbb{C}^{(n+m)\times(n+m)}$, $\mathbf{\Delta} = \{\delta I_m : \delta \in \mathbb{C}\}$,
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(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
$\max_{\delta_2 \in \overline{\mathbb{D}}} \bar{\sigma}(\sqrt{P}\mathscr{S}(M, \delta_2 I_m)\sqrt{P}^{-1}) < 1.$

(2) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \max_{\delta_2 \in \overline{\mathbb{D}}} \bar{\sigma}(\mathscr{S}(M^P, \delta_2 I_m)) < 1.$

(3) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \mu_{\mathbf{\Delta}_S}(M^P) < 1.$

11.3. Complex state space data, 1 repeated complex perturbation

Let $M \in \mathbb{C}^{(n+m)\times(n+m)}$, $\mathbf{\Delta} = \{\delta I_m : \delta \in \mathbb{C}\}$,
$\mathbf{\Delta}_S := \{\text{diag}[\triangle_1, \delta_2 I_m] : \triangle_1 \in \mathbb{C}^{n\times n}, \delta_2 \in \mathbb{C}\}$. Then are equivalent

(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
$\max_{\delta_2 \in \overline{\mathbb{D}}} \ \bar{\sigma}(\sqrt{P}\mathscr{S}(M, \delta_2 I_m)\sqrt{P}^{-1}) < 1.$

(2) $\inf_{P \in \mathbb{C}^{n\times n},\ P = P^* > 0} \ \max_{\delta_2 \in \overline{\mathbb{D}}} \ \bar{\sigma}(\mathscr{S}(M^P, \delta_2 I_m)) < 1.$

(3) $\inf_{P \in \mathbb{C}^{n\times n},\ P = P^* > 0} \ \mu_{\mathbf{\Delta}_S}(M^P) < 1.$

(4) $\inf_{P \in \mathbb{C}^{n\times n},\ P = P^* > 0,\ D_2 \in \mathbb{C}^{n\times n},\ D_2 = D_2^* > 0}$
$\bar{\sigma}\left(\begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2} \end{bmatrix} M^P \begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2}^{-1} \end{bmatrix}\right) < 1.$

11.3. Complex state space data, 1 repeated complex perturbation

Let $M \in \mathbb{C}^{(n+m)\times(n+m)}$, $\mathbf{\Delta} = \{\delta I_m : \delta \in \mathbb{C}\}$, $\mathbf{\Delta}_S := \{\mathrm{diag}[\triangle_1, \delta_2 I_m] : \triangle_1 \in \mathbb{C}^{n\times n}, \delta_2 \in \mathbb{C}\}$. Then are equivalent

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$\max_{\delta_2 \in \bar{\mathbb{D}}} \bar{\sigma}(\sqrt{P}\mathscr{S}(M, \delta_2 I_m)\sqrt{P}^{-1}) < 1.$

(2) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \max_{\delta_2\in\bar{\mathbb{D}}} \bar{\sigma}(\mathscr{S}(M^P, \delta_2 I_m)) < 1.$

(3) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \mu_{\mathbf{\Delta}_S}(M^P) < 1.$

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$$\bar{\sigma}\left(\begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2}\end{bmatrix} M^P \begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2}^{-1}\end{bmatrix}\right) < 1.$$

(5) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ D_2\in\mathbb{C}^{m\times m},\ D_2=D_2^*>0}$
$$\bar{\sigma}\left(\begin{bmatrix} \sqrt{P} & 0 \\ 0 & \sqrt{D_2}\end{bmatrix} M \begin{bmatrix} \sqrt{P}^{-1} & 0 \\ 0 & \sqrt{D_2}^{-1}\end{bmatrix}\right) < 1.$$

11.3. Complex state space data, 1 repeated complex perturbation

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$\max_{\delta_2 \in \bar{\mathbb{D}}} \bar{\sigma}(\sqrt{P}\mathscr{S}(M, \delta_2 I_m)\sqrt{P}^{-1}) < 1.$

(2) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \max_{\delta_2\in\bar{\mathbb{D}}} \bar{\sigma}(\mathscr{S}(M^P, \delta_2 I_m)) < 1.$

(3) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \mu_{\mathbf{\Delta}_S}(M^P) < 1.$

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$$\bar{\sigma}\left(\begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2}\end{bmatrix} M^P \begin{bmatrix} I_n & 0 \\ 0 & \sqrt{D_2}^{-1}\end{bmatrix}\right) < 1.$$

(5) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ D_2\in\mathbb{C}^{m\times m},\ D_2=D_2^*>0}$
$$\bar{\sigma}\left(\begin{bmatrix} \sqrt{P} & 0 \\ 0 & \sqrt{D_2}\end{bmatrix} M \begin{bmatrix} \sqrt{P}^{-1} & 0 \\ 0 & \sqrt{D_2}^{-1}\end{bmatrix}\right) < 1.$$

(6) $\rho(M_{11}) < 1$ and $\inf_{D_2\in\mathbb{C}^{m\times m},\ D_2=D_2^*>0} \|\sqrt{D_2}G\sqrt{D_2}^{-1}\|_\infty < 1.$

Moreover, quadratic stability w.r.t. $\mathscr{S}(M, \delta I_m)$ is equivalent to non-emptyness of the convex set

$$\left\{X = \begin{bmatrix} P & 0 \\ 0 & D_2 \end{bmatrix} : P \in \mathbb{C}^{n\times n}, D_2 \in \mathbb{C}^{m\times m}, X = X^* > 0, M^*XM - X < 0\right\}$$

11.4. Complex state space data, 2 full complex blocks

Let $M \in \mathbb{C}^{(n+m)\times(n+m)}$,

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(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
$\max_{\triangle \in \mathbf{B_\Delta}} \ \bar{\sigma}(\sqrt{P}\mathscr{S}(M, \triangle)\sqrt{P}^{-1}) < 1.$

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(3) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \ \mu_{\mathbf{\Delta}_C}(M^P) < 1.$

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(1) $\exists P \in \mathbb{C}^{n\times n}$, $P = P^* > 0$:
$\max_{\triangle\in\mathbf{B}_{\mathbf{\Delta}}} \ \bar{\sigma}(\sqrt{P}\mathscr{S}(M,\triangle)\sqrt{P}^{-1}) < 1.$

(2) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \ \max_{\triangle\in\mathbf{B}_{\mathbf{\Delta}}} \ \bar{\sigma}(\mathscr{S}(M^P,\triangle)) < 1.$

(3) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0} \ \mu_{\mathbf{\Delta}_C}(M^P) < 1.$

(4) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ d_1,d_2>0}$

$$\bar{\sigma}\left(\begin{bmatrix} \sqrt{d_1}I_n & 0 & 0 \\ 0 & \sqrt{d_2}I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix} M^P \begin{bmatrix} \sqrt{d_1}^{-1}I_n & 0 & 0 \\ 0 & \sqrt{d_2}^{-1}I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix}\right)$$
$< 1.$

(5) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ d_1,d_2>0}$
$$\bar{\sigma}\left(\begin{bmatrix}\sqrt{d_1}\sqrt{P} & 0 & 0\\ 0 & \sqrt{d_2}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix} M \begin{bmatrix}\sqrt{d_1}^{-1}\sqrt{P}^{-1} & 0 & 0\\ 0 & \sqrt{d_2}^{-1}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix}\right)$$
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$< 1.$

(6) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ d_2>0}$
$$\bar{\sigma}\left(\begin{bmatrix}\sqrt{P} & 0 & 0\\ 0 & \sqrt{d_2}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix} M \begin{bmatrix}\sqrt{P}^{-1} & 0 & 0\\ 0 & \sqrt{d_2}^{-1}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix}\right)$$
$< 1.$

(5) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ d_1,d_2>0}$

$$\bar{\sigma}\left(\begin{bmatrix}\sqrt{d_1}\sqrt{P} & 0 & 0\\ 0 & \sqrt{d_2}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix} M \begin{bmatrix}\sqrt{d_1}^{-1}\sqrt{P}^{-1} & 0 & 0\\ 0 & \sqrt{d_2}^{-1}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix}\right)$$

$< 1.$

(6) $\inf_{P\in\mathbb{C}^{n\times n},\ P=P^*>0,\ d_2>0}$

$$\bar{\sigma}\left(\begin{bmatrix}\sqrt{P} & 0 & 0\\ 0 & \sqrt{d_2}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix} M \begin{bmatrix}\sqrt{P}^{-1} & 0 & 0\\ 0 & \sqrt{d_2}^{-1}I_{m_1} & 0\\ 0 & 0 & I_{m_2}\end{bmatrix}\right)$$

$< 1.$

(7) $\rho(M_{11}) < 1$ and

$$\inf_{d_2>0}\left\|\begin{bmatrix}\sqrt{d_2}I_{m_1} & 0\\ 0 & I_{m_2}\end{bmatrix} G \begin{bmatrix}\sqrt{d_2}^{-1}I_{m_1} & 0\\ 0 & I_{m_2}\end{bmatrix}\right\|_\infty < 1.$$