On the failure of the Denjoy-Wolff theorem in bounded convex domains

Filippo Bracci (*Tor Vergata, Rome*)

In this talk, based on a work in progress with Yekta Okten, I will discuss the Denjoy-Wolff theorem in bounded convex domains. More precisely, let's say that a bounded domain D has the Denjoy-Wolff property if the iterates of any holomorphic self-map of D without fixed points converge to a boundary point. Several authors worked on the question of characterizing bounded convex domains which have the Denjoy-Wolff property. For instance, it is known that if D is sufficiently smooth and linearly strongly convex (Abate, Abate and Raissy), or if D is Gromov hyperbolic with respect to the Kobayashi distance (Gaussier, Zimmer and the speaker), or, more generally, if it is visible (Bharali-Maitra) then it has the Denjoy-Wolff property. Visibility in general is just a sufficient condition for the Denjoy-Wolff property, but it is not necessary, as one can construct a bounded simply connected (not convex) domain in $\mathbb C$ with the Denjoy–Wolff property but failing to be visible. However, I conjectured some years ago that for bounded convex domains visibility is equivalent to the Denjoy-Wolff property. The conjecture is still open, mainly because it is rather hard to characterize geometrically bounded convex domains which are visible. In this talk I will present a construction which shows that if a bounded convex domain D has a real segment in the boundary which is contained in an affine disc (a "real-complex segment") and contains an "edge" with sufficiently large aperture having such a real-complex segment on the boundary, then D does not have the Denjoy-Wolff property. This in particular applies to all bounded convex domains containing an analytic disc E on the boundary. In this latter case I will also show that the target set (the set of accumulation points of the iterates of a holomorphic self-map) can be essentially whatever connected picture in E. I will also discuss some ideas about geometric features of visibility and Denjoy-Wolff property in convex domains.