Function theory off the complexified unit circle

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We discuss joint work with Annika Moucha, Oliver Roth and Toshiyuki Sugawa. The central object of this talk is the algebra of holomorphic functions on

$$\Omega := \left\{ (z,w) \in \widehat{\mathbb{C}}^2 \colon zw \neq 1 \right\},$$

which may be understood as the complement of the "complexified unit circle" and constitutes an open submanifold of $\widehat{\mathbb{C}}^2$. Remarkably, Ω provides a unifying framework for the study of conformally equivariant differential operators, the spectral theory of the Riemannian Laplacians and strict deformation quantization of the unit disk and Riemann sphere. Both domains are contained faithfully within Ω as diagonals, and already determine the behaviour of holomorphic objects globally by a version of the identity principle. It should thus not be too surprising that Ω arises naturally also in other contexts: within Lie theory as the *crown domain* of the special linear group, as the *second configuration space* of the Riemann sphere in the context of many-body mechanics, and algebra-geometrically as the *complexified two-sphere*. We showcase the resulting machinery by constructing a continuous Wick-type star product $\star_\hbar$ on $\mathcal{H}(\Omega)$ by means of Peschl–Minda differential operators. It depends meromorphically on a complex parameter $\hbar$ and, unlike formal deformation quantizations, is given by a factorial series. Finally, we establish that its formal analogue provides an asymptotic expansion in powers of $\hbar$.