Research group: Invariant Metrics in Complex Analysis
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Abstracts

Julia–Wolff–Carathéodory theorem and pluricomplex Poisson kernel in convex domains of finite type

Leandro Arosio
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The classical Julia–Wolff–Carathéodory theorem shows that, if f is a holomorphic self-map of the disc, the derivative f' admits a positive nontangential limit near any boundary regular fixed point z, and the limit equals the dilation of f at z which can be computed in terms of the Poincaré distance. This result had several generalizations to several variables: in particular Rudin proved a version of it in the ball, Abate in strongly convex domains, and Abate–Tauraso in convex domains of d'Angelo finite type, adding a couple of technical assumptions. In this talk I will show how to prove the full theorem in the context of convex domains of d'Angelo finite type, using the strong asymptoticity of complex geodesics and the existence of horospheres. This result turns out to be related to the pluricomplex Poisson kernel introduced by Bracci–Patrizio–Trapani.

This is based on joint works with Matteo Fiacchi and Filippo Bracci.

Local continuous extension of proper holomorphic maps: low-regularity and infinite-type boundaries

Annapurna Banik
(*Tata Institute of Fundamental Research–Centre for Applicable Mathematics, Bangalore*)

We shall discuss a couple of results on local continuous extension of proper holomorphic maps $F : D \longrightarrow \Omega$, $D, \Omega \subsetneq \mathbb{C}^n$, making local assumptions on their boundaries ∂D and $\partial\Omega$. The first result allows us to have much lower regularity, for the patches of $\partial D, \partial\Omega$ that are relevant, than in earlier results in the literature. The second result is in the spirit of a result by Forstnerič–Rosay. However, our assumptions allow $\partial\Omega$ to contain boundary points of infinite type.

In this talk, we will first discuss the motivations for the above results. We shall also discuss the key ideas behind the proofs of these results.

On the failure of the Denjoy–Wolff theorem in bounded convex domains

Filippo Bracci
(*Tor Vergata, Rome*)

In this talk, based on a work in progress with Yekta Okten, I will discuss the Denjoy–Wolff theorem in bounded convex domains. More precisely, let's say that a bounded domain D has the Denjoy–Wolff property if the iterates of any holomorphic self-map of D without fixed points converge to a boundary point. Several authors worked on the question of characterizing bounded convex domains which have the Denjoy–Wolff property. For instance, it is known that if D is sufficiently smooth and linearly strongly convex (Abate, Abate and Raissy), or if D is Gromov hyperbolic with respect to the Kobayashi distance (Gaussier, Zimmer and the speaker), or, more generally, if it is visible (Bharali–Maitra) then it has the Denjoy–Wolff property. Visibility in general is just a sufficient condition for the Denjoy–Wolff property, but it is not necessary, as one can construct a bounded simply connected (not convex) domain in $\mathbb{C}$ with the Denjoy–Wolff property but failing to be visible. However, I conjectured some years ago that for bounded convex domains visibility is equivalent to the Denjoy–Wolff property. The conjecture is still open, mainly because it is rather hard to characterize geometrically bounded convex domains which are visible. In this talk I will present a construction which shows that if a bounded convex domain D has a real segment in the boundary which is contained in an affine disc (a "real-complex segment") and contains an "edge" with sufficiently large aperture having such a real-complex segment on the boundary, then D does not have the Denjoy–Wolff property. This in particular applies to all bounded convex domains containing an analytic disc E on the boundary. In this latter case I will also show that the target set (the set of accumulation points of the iterates of a holomorphic self-map) can be essentially whatever connected picture in E. I will also discuss some ideas about geometric features of visibility and Denjoy–Wolff property in convex domains.

Holomorphicity of Kobayashi isometry

Anand Chavan
(*Jagiellonian University, Kraków*)

In this talk we will discuss the problem of holomorphicity of Kobayashi isometry. Given a isometry between two domains in complex Euclidean space with respect to their Kobayashi distance/metric, it is an interesting problem to know when is this isometry holomorphic. We will see through few examples that Kobayashi isometry need not be holomorphic and mention some important results in this context. In the end we will show for the domain diamond $\triangle = \{|z_1| + |z_2| < 1\} \subset \mathbb{C}^2$ and special Carathéodory sets of tridisc $D_{a,b} = \{(z, w) \in \mathbb{D}^2 : |az_1 + bz_2 - z_1z_2| < |az_2 + bz_1 - 1|\}$ for $a, b > 0$ the Kobayashi isometry is holomorphic.

New projections and duality in Bergman spaces

Luke Edholm
(*University of Vienna*)

Given a domain $\Omega \subset \mathbb{C}^n$ and $p > 1$, let $A^p(\Omega)$ denote the L^p-Bergman space, the holomorphic subspace of $L^p(\Omega)$. When Ω is smoothly bounded and strongly pseudoconvex, it is well known that the dual of $A^p(\Omega)$ can be naturally identified with $A^q(\Omega)$, where $p^{-1} + q^{-1} = 1$. This follows the established paradigm seen in ordinary L^p-spaces, and is closely linked to the L^p-mapping regularity of the Bergman projection.

The presence of boundary singularities can cause the above dual space characterization to fail. In this talk, we look at this question on monomial polyhedra, a class of non-smooth and weakly pseudoconvex domains in $\mathbb{C}^n$, where the L^p-regularity of the Bergman projection and the A^p-A^q duality paradigm both break down. We will construct a family of new projection operators with better L^p-mapping behavior than the Bergman projection, then use them to concretely characterize the duals of A^p-spaces on these domains.

Holomorphicity of isometries in IV classical domains (Lie balls)

Armen Edigarian
(*Jagiellonian University, Kraków*)

We show that any $\mathcal{C}^1$ isometry in the sense of Kobayashi–Royden metric between IV classical domains is holomorphic or anti-holomorphic.

Gromov hyperbolicity and precise estimates for certain distances in $\mathbb{R}^d$

Matteo Fiacchi
(*Tor Vergata, Rome*)

In the first part of the talk, we will prove that strongly minimally convex domains are Gromov hyperbolic with respect to the minimal metric by establishing estimates similar to those of Balogh and Bonk for the Kobayashi metric in strongly pseudoconvex domains. In the second part, we will improve previous estimates for certain Finsler metrics in domains of $\mathbb{R}^d$. These metrics include the Kobayashi-Hilbert metric near strongly convex points, the minimal metric near convex and strongly minimally convex points, and the k-quasi-hyperbolic metric in k-strongly convex domains.

The second part of the talk is based on joint work with N. Nikolov.

Function theory off the complexified unit circle

Michael Heins
(*Würzburg University*)

We discuss joint work with Annika Moucha, Oliver Roth and Toshiyuki Sugawa. The central object of this talk is the algebra of holomorphic functions on

$$\Omega := \left\{(z,w) \in \widehat{\mathbb{C}}^2 \colon zw \neq 1\right\},$$

which may be understood as the complement of the "complexified unit circle" and constitutes an open submanifold of $\widehat{\mathbb{C}}^2$. Remarkably, Ω provides a unifying framework for the study of conformally equivariant differential operators, the spectral theory of the Riemannian Laplacians and strict deformation quantization of the unit disk and Riemann sphere. Both domains are contained faithfully within Ω as diagonals, and already determine the behaviour of holomorphic objects globally by a version of the identity principle. It should thus not be too surprising that Ω arises naturally also in other contexts: within Lie theory as the *crown domain* of the special linear group, as the *second configuration space* of the Riemann sphere in the context of many-body mechanics, and algebra-geometrically as the *complexified two-sphere*. We showcase the resulting machinery by constructing a continuous Wick-type star product $\star_\hbar$ on $\mathcal{H}(\Omega)$ by means of Peschl–Minda differential operators. It depends meromorphically on a complex parameter $\hbar$ and, unlike formal deformation quantizations, is given by a factorial series. Finally, we establish that its formal analogue provides an asymptotic expansion in powers of $\hbar$.

Weighted Szegö kernels

Aakanksha Jain
(*Jagiellonian University, Kraków*)

We shall talk about the weighted Szegö kernels on planar domains and see some examples. Results regarding the variation of the weighted Szegö kernel and some of its consequences will be discussed. We shall explore the question of the zeroes of the weighted Szegö kernels and see its relation with other reproducing kernels.

Visibility domains in complex manifolds

Rumpa Masanta
(*Indian Institute of Science, Bangalore*)

In this talk, we extend the notion of visibility with respect to the Kobayashi distance to domains in arbitrary complex manifolds. Visibility here is a weak notion of negative curvature and refers to a property resembling visibility in the sense of Eberlein–O'Neill for Riemannian manifolds. However, we do not assume Cauchy-completeness, with respect to the Kobayashi distance, of the domains in question. The visibility property of a domain D can be used to deduce many properties of certain holomorphic mappings into D, ranging from their continuous extendibility to the iterative dynamics of such self-maps of D. Here, we present a few sufficient conditions for visibility in the above setting, and with these conditions, we see that the class of domains with the visibility property is very large. Finally, we will discuss an application of visibility by discussing some generalizations of the classical Wolff–Denjoy theorem.

Carathéodory sets in the polydisk

John E. McCarthy
(*Washington University, St. Louis*)

The set

$$\mathcal{K} = \{(x,y,z) \in \mathbb{D}^3 : x+y+z = xy+yz+xz\}$$

originally appeared as the uniqueness set for a 3 point extremal Pick problem on the tridisk [Ł. Kosiński, *Three-point Nevanlinna-Pick problem in the polydisc*, Proc. Lond. Math. Soc. (3) **111** (2015), no. 4, 887–910]. Later Kosiński and Zwonek proved that $\mathcal{K}$ is a Carathéodory set for $\mathbb{D}^3$, i.e. that its intrinsic Carathéodory metric agrees with the Carathéodory metric of the tridisk [Ł. Kosiński and W. Zwonek, *Extension property and universal sets*, Canad. J. Math. **73** (2021), no. 3, 717–736].

It turns out that every Carathéodory set in the polydisk is built out of copies of $\mathcal{K}$ and $\mathbb{D}$. We shall describe how this happens.

This is ongoing joint work with Ł. Kosiński.

Maximal conformal metrics and bounded holomorphic functions

Annika Moucha
(*Würzburg University*)

Denote by $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ the open unit disk. Maximal Blaschke products are those holomorphic self-maps of $\mathbb{D}$ that are determined by their critical points, i.e. the zeros of its derivative, in the following sense: D. Kraus and O. Roth proved that if $\mathcal{C}$ is a set of critical points of a nonconstant holomorphic self-map of $\mathbb{D}$, then there exists a Blaschke product B such that the set of critical points of B is precisely $\mathcal{C}$ and

$$\frac{|f'(z)|}{1-|f(z)|^2} \leq \frac{|B'(z)|}{1-|B(z)|^2}, \quad z \in \mathbb{D},$$

whenever $f : \mathbb{D} \longrightarrow \mathbb{D}$ is a holomorphic function having critical points (at least) $\mathcal{C}$. From the point of view of hyperbolic geometry the above result can be understood as a sharpening of the classical Ahlfors lemma for conformal pseudometrics with predescribed zeros: the conformal pseudometric $\lambda_B|dz|$ that is obtained from the pullback of the hyperbolic metric under B, i.e. $\lambda_B(z) = |B'(z)|/(1-|B(z)|^2)$, is maximal among all conformal pseudometrics on $\mathbb{D}$ with zero set (at least) $\mathcal{C}$ and curvature (in fact even bounded above by) -4.

In this talk we present some recent results about maximal Blaschke products such as boundary rigidity and forward iteration.

Weak triangle inequality for the Lempert function

Nikolai Nikolov
(*Bulgarian Academy of Science, Sofia*)

The (unbounded version of the) Lempert function l_D on a domain $D \subset \mathbb{C}^d$ does not usually satisfy the triangle inequality, even for smooth balanced domains. However, on bounded $\mathcal{C}^2$-smooth strictly pseudoconvex domains, it satisfies a weaker version with a constant: $l_D(a,c) \leq C(l_D(a,b) + l_D(b,c))$. We show that pseudoconvexity is necessary for this property as soon as D has a $\mathcal{C}^1$-smooth boundary. We also give some estimates in some domains which are model for local situations.

Based on a joint work with Pascal J. Thomas.

Extension of isometries of the Kobayashi metric and localization

Amar Deep Sarkar
(*Indian Institute of Technology, Bhubaneswar*)

Recently, there have been many applications of visibility with respect to the Kobayashi distance in several complex variables, such as the extension of biholomorphisms (Kobayashi isometries) and Wolff–Denjoy type theorems. Furthermore, it has been observed that weak visibility can be used to localize the Kobayashi distance. In this talk, we will discuss some applications of weak visibility and visibility.

We begin by introducing visibility and weak visibility with respect to the Kobayashi distance. Next, we discuss the extension of a biholomorphic map between two domains in complex Euclidean space. To prove this extension phenomenon, we rely on the localization of the Kobayashi distance near visible boundary points. We also explore this localization result, which connects the local and global Kobayashi distances of a domain in the complex Euclidean space.