Carathéodory sets in the polydisk

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The set

$$\mathcal{K} = \{(x, y, z) \in \mathbb{D}^3 : x + y + z = xy + yz + xz\}$$

originally appeared as the uniqueness set for a 3 point extremal Pick problem on the tridisk [Ł. Kosiński, *Three-point Nevanlinna-Pick problem in the polydisc*, Proc. Lond. Math. Soc. (3) **111** (2015), no. 4, 887–910]. Later Kosiński and Zwonek proved that $\mathcal{K}$ is a Carathéodory set for $\mathbb{D}^3$, i.e. that its intrinsic Carathéodory metric agrees with the Carathéodory metric of the tridisk [Ł. Kosiński and W. Zwonek, *Extension property and universal sets*, Canad. J. Math. **73** (2021), no. 3, 717–736].

It turns out that every Carathéodory set in the polydisk is built out of copies of $\mathcal{K}$ and $\mathbb{D}$. We shall describe how this happens.

This is ongoing joint work with Ł. Kosiński.