Maximal conformal metrics and bounded holomorphic functions

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Denote by $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ the open unit disk. Maximal Blaschke products are those holomorphic self-maps of $\mathbb{D}$ that are determined by their critical points, i.e. the zeros of its derivative, in the following sense: D. Kraus and O. Roth proved that if $\mathcal{C}$ is a set of critical points of a nonconstant holomorphic self-map of $\mathbb{D}$, then there exists a Blaschke product B such that the set of critical points of B is precisely $\mathcal{C}$ and

$$\frac{|f'(z)|}{1-|f(z)|^2} \leq \frac{|B'(z)|}{1-|B(z)|^2}, \quad z \in \mathbb{D},$$

whenever $f : \mathbb{D} \longrightarrow \mathbb{D}$ is a holomorphic function having critical points (at least) $\mathcal{C}$. From the point of view of hyperbolic geometry the above result can be understood as a sharpening of the classical Ahlfors lemma for conformal pseudometrics with predescribed zeros: the conformal pseudometric $\lambda_B|dz|$ that is obtained from the pullback of the hyperbolic metric under B, i.e. $\lambda_B(z) = |B'(z)|/(1-|B(z)|^2)$, is maximal among all conformal pseudometrics on $\mathbb{D}$ with zero set (at least) $\mathcal{C}$ and curvature (in fact even bounded above by) -4.

In this talk we present some recent results about maximal Blaschke products such as boundary rigidity and forward iteration.