Local Spectral Theory for Normal Operators in Krein Spaces

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Abstract

Let $N$ be a bounded normal operator in a Krein space $(\mathcal{H}, [\cdot, \cdot])$. Recall that a bounded operator $N$ in a Krein space is normal if $NN^+ = N^+N$, where $N^+$ denotes the adjoint operator of $N$ with respect to $[\cdot, \cdot]$. A point $\lambda$ of the approximative point spectrum $\sigma_{ap}(N)$ of $N$ is called a spectral point of positive (negative) type, if for every normed approximative eigensequence $(x_n)$ corresponding to $\lambda$ all accumulation points of the sequence $([x_n, x_n])$ are positive (resp. negative).

The spectral theory for normal operators in Krein spaces is a rather underdeveloped subject and most results are based on the definitizability of the real and the imaginary part of $N$. We will present a somewhat different approach.

We investigate bounded normal operators in Krein spaces having a real part and an imaginary part with real spectra only. If moreover the imaginary part satisfies some growth condition close to the real axis and if there is a rectangular consisting of either positive or negative type spectrum only, then the normal operator possesses a local spectral function which is defined for Borel subsets of the rectangular, i.e. the operator behaves locally in the same way as a normal operator in a Hilbert space.

This talk is based on a joint work with F. Philipp (Ilmenau, Germany) and V. Strauss (Caracas, Venezuela).

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