

# On the group structure of automorphism groups of $G$ -manifolds with codimension one orbit

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Let  $G$  be a compact Lie group and  $\mathcal{L}_G(M)$  the group of equivariant Lipschitz homeomorphisms of a smooth  $G$ -manifold  $M$  with compact support.  $L_G(M)$  and  $\mathcal{H}_{LIP,G}(M)$  denote the identity component of  $\mathcal{L}_G(M)$  with respect to the compact open topology and the compact open Lipschitz topology, respectively. In this talk first we describe the following.

**Theorem 1** ([A-F2]) *Let  $V$  be a  $G$ -module with codimension one orbit. Then the group  $\mathcal{H}_{LIP,G}(V)$  is perfect.*

In [A-F-M] we proved that  $L_{U(n)}(\mathbb{C}^n)$  is not perfect if  $\mathbb{C}^n$  has the canonical  $U(n)$ -action. In fact the first homology group  $H_1(L_{U(n)}(\mathbb{C}^n))$  is isomorphic to a real valued function space on  $(0, 1]$  which admits continuous moduli. In [A-F1] we studied the group  $\mathcal{D}_G(M)$  of equivariant diffeomorphisms of a smooth  $G$ -manifold  $M$  which are  $G$ -isotopic to the identity through equivariant diffeomorphisms with compact support. Then we proved that  $H_1(\mathcal{D}_{U(n)}(\mathbb{C}^n)) \cong U(1) \times \mathbb{R}$ .

Therefore the first homology groups of the automorphism groups of a  $G$ -manifold  $M$  are quite depend on the category of the automorphism group.

Secondary we consider the case of  $M$  to be a smooth connected closed  $G$ -manifold with codimension one orbit. Let  $(H)$  be the principal orbit type of  $M$  and  $(K_0), (K_1)$  be the singular orbit types of  $M$ . Let  $N(H)$  be the normalizer of the group  $H$  in  $G$ . Analysing the behavior of Lipschitz homeomorphisms around the singular orbits and using Theorem 1 we have.

**Theorem 2** ([A-F2])  $H_1(\mathcal{H}_{LIP,G}(M)) \cong H_1(\bar{W}(M))$ .

Here

$$\bar{W}(M) = \left( \frac{N(H) \cap N(K_0)}{K_0} \times \frac{N(H) \cap N(K_1)}{K_1} \right)_0.$$

## References

- [A-F1] K. Abe and K. Fukui, *On the structure of the group of equivariant diffeomorphisms of  $G$ -manifolds with codimension one orbit*, *Topology*, 40 (2001), 1325-1337.
- [A-F2] K. Abe and K. Fukui, *On the first homology of the group of Lipschitz homeomorphisms of  $G$ -manifolds with codimension one orbit*, preprint (2008).
- [A-F-M] K. Abe, K. Fukui and T. Miura, *On the first homology of the group of equivariant Lipschitz homeomorphisms*, *J. Math. Soc. Japan*, 58 (2006), 1-15.