Geometry of the Schrödinger operator

Janusz Grabowski

The Schrödinger operators on the Newtonian space-time are defined in a way which make them independent on the class of inertial observers. In this picture the Schrödinger operators act not on functions on the space-time but on sections of certain one-dimensional complex vector bundle – the *Schrödinger line bundle*. This line bundle has trivializations indexed by inertial observers and is associated with an $U(1)$-principal bundle with an analogous list of trivializations – the *Schrödinger principal bundle*. If an inertial frame is fixed, the Schrödinger bundle can be identified with the trivial bundle over space-time, but as there is no canonical trivialization (inertial frame), these sections interpreted as 'wave-functions' cannot be viewed as actual functions on the space-time. In this approach the change of an observer results not only in the change of actual coordinates in the space-time but also in a change of the phase of wave functions. For the Schrödinger principal bundle a natural differential calculus for 'wave forms' is developed that leads to a natural generalization of the concept of Laplace-Beltrami operator associated with a pseudo-Riemannian metric. The free Schrödinger operator turns out to be the Laplace-Beltrami operator associated with a naturally distinguished invariant pseudo-Riemannian metric on the Schrödinger principal bundle. The presented framework does not involve any *ad hoc* or axiomatically introduced geometrical structures. It is based on the traditional understanding of the Schrödinger operator in a given reference frame – which is supported by producing right physics predictions – and it is proven to be strictly related to the frame-independent formulation of analytical Newtonian mechanics and Hamilton-Jacobi equations, that makes a bridge between the classical and quantum theory.