On topological classification of Morse-Smale diffeomorphisms

Viatcheslav Grines

The report is devoted to exposition of results obtained by the author in collaboration with E. Gurevuch and V. Medvedev. Let M^n is the smooth closed orientable manifold of dimension $n \geq 4$. We consider class $G_1(M^n)$ of orientation preserving Morse-Smale diffeomorphisms of M^n, such that for any $f \in G_1(M^n)$ the non-wandering set $\Omega(f)$ have a next properties:

1) the set of saddle periodic point of f is not empty and all of them have Morse index equal to one.

2) unstable and stable manifolds of different saddle periodic points from $\Omega(f)$ have no intersection.

The purpose of the talk to present a topological classification of the diffeomorphisms from the $f \in G_1(M^n)$. Note at once that, for $n = 2$, the solution of this problem follows from [1] and is a generalization of classical results on the classification of structurally stable flows on surfaces. The topological invariants in this case are immaterial generalizations of a scheme suggested by E.A. Leontovich-Andronova and A.G. Maier and the M. Peixoto graph. Comparatively recently, it has turned out that the case $n = 3$ differs essentially from $n = 2$. For example, in [2], it was shown that the class of diffeomorphisms of the 3-sphere with nonwandering set consisting of four fixed points contains a countable set of topologically nonconjugate diffeomorphisms (although all diffeomorphisms have the same Peixoto graph). This effect turned out to be related to the possibility of a wild embedding of the closure of the separatrices of saddle periodic points in the ambient manifold, which leads to the necessity of introducing new invariants for describing such a situation. Examples of wild embeddings of curves and two-dimensional spheres were constructed by J. Alexander in 1924 and by E. Artin and R. Fox in 1948. However, only in 1977, D. Pixton discovered that such examples can be closures of invariant manifolds of structurally stable diffeomorphisms. Topological invariants for Morse-Smale diffeomorphisms defined on 3-manifolds under assumptions of various generality were constructed by Ch. Bonatti, E. Pecou, V.Z. Grines, V.S. Medvedev, and O.V. Pochinka in series of papers in 2000–2005. (see for example [3] for results and references).

In this talk we show that, for dimension higher than 3, the oriented Peixoto graph with a given permutation on its vertices is a complete topological invariant (see [4]). The proof of this result essentially uses facts of multidimensional topology, some of them do not hold in dimension 3.

References

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[2] Ch. Bonatti, V. Grines, *Knots as topological invariant for gradient-like diffeomorphisms of the sphere S^3.* Journal of Dynamical and Control Systems. No 6 (2000), 579-602.

[3] Ch. Bonatti, V.Z. Grines, O.V. Pochinka. *The classification of Morse-Smale diffeomorphisms with finit set of geteroclinic orbits on 3-manifolds.* Trudy Steklov institute v. 250 (2005),

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[4] V. Z. Grines, E. Ya. Gurevich. *On MorseSmale Diffeomorphisms on Manifolds of Dimension Higher than Three.* Doklady Mathematics, Vol. 76, No. 2 (2007), 649-651.