Yang-Mills bar connections over compact Kähler manifolds

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Let M be a compact Hermitian manifold and E a Hermitian vector bundle over M. The Koszul-Malgrange criterion asserts that E carries a holomorphic structure, if and only if there is a unitary connection A on E such that the $(0,2)$-component $F_A^{0,2}$ of the curvature F_A of A vanishes. Thus we shall call a unitary connection A satisfying the Koszul-Malgrange criterion a unitary holomorphic connection. We introduce a Yang-Mills bar equation as the Euler-Lagrangian equation for the Yang-Mills bar functional which is equal to the L_2-norm of the $(0,2)$-component $F_A^{0,2}$ of a unitary connection A. This equation has an advantage over the equation for a holomorphic connection because the latter one is overdetermined and the first one is elliptic modulo a degeneracy which is formally generated by the action of the complex gauge group. Solutions of the Yang-Mills bar equation are called Yang-Mills bar connections. Among Yang-Mills bar connections over compact Kähler manifolds there is a distinguish class of quasi-holomorphic connections which include unitary holomorphic connections. Einstein quasi-holomorphic connections are introduced as a direct generalization of the notion of Hermitian Yang Mills connections. We show the existence of non-trivial Einstein quasi-holomorphic connections over torus. We show that any Einstein quasi-holomorphic connection on a Hermitian vector bundle over a compact Kähler manifold with positive Ricci curvature is in fact a holomorphic connection (and hence Hermitian Yang Mill connection). We also discuss the existence of the negative Yang-Mill bar gradient flow.