Commutators of the C^r-diffeomorphism groups on a manifold with corners

Jacek Lech

A group G is perfect iff $G = [G, G]$, where the commutator subgroup is generated by elements $fgf^{-1}g^{-1}$, $f, g \in G$. In terms of homology of groups it means that $H_1(G) = G/[G, G] = 0$.

Let M be a manifold of dimension n. We denote by $\mathrm{Diff}^r_c(M)_0$ the identity component of the group of compactly supported C^r-diffeomorphisms on M. It is well known from theorems of Herman, Thurston and Mather that the group $\mathrm{Diff}^r_c(M)_0$ on a standard manifold M is perfect and simple, where $r = 0, 1, \dots, \infty$, $r \neq n+1$. Rybicki proved an analogous result for a manifold with boundary.

In the presented proof we exploit Mather-Epstein's method to show the perfectness theorem for a manifold with corners. The idea of the proof is to use Mather's operator once, along some edge of M. Next we introduce a bump decomposition, which means a special kind of fragmentation using bump functions. Then we may prove that the group $\mathrm{Diff}^r_c(M)_0$ is perfect for $n \geq 2$ and $r = n+1, \dots, \infty$, whenever M has no vertices. The last assumption follows from the fact that the Mather's operator cannot be used transversally to the edges. In the case of a manifold with vertices we get from Fukui's results that $\mathrm{Diff}^r_c(M)_0$ is not perfect. Especially we have $H_1(\mathrm{Diff}^r_c([0, \infty))_0) = \mathbb{R}$ and $H_1(\mathrm{Diff}^r_c([0, 1])_0) = \mathbb{R}^2$.