Functions with isolated singularities on surfaces

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Let M be a smooth connected compact surface, P be either a real line $\mathbb{R}$ or a circle S^1, and $f: M \to P$ a smooth mapping. Denote by Σ_f the set of critical points of f. Let also $\mathcal{S}(f)$ and $\mathcal{O}(f)$ be the stabilizer and orbit of f under the action of the identity component $\mathrm{Diff}_{\mathrm{id}}(M)$ of the group of diffeomorphisms of M. We introduce four axioms for f under which one is able to describe the homotopy types of $\mathcal{S}$ and $\mathcal{O}$. This result extends the analogous calculations concerning Morse functions [**?**, **?**] and is based on the results of [**?**, **?**].

In particular, suppose that $f: M \to P$ has the following properties:

(1) f is constant on every connected component of ∂M and $\Sigma_f \subset Int(M)$;

(2) for every critical point $z \in \Sigma_f$ there is a local presentation $f: \mathbb{R}^2 \to \mathbb{R}$ of f such that $z = (0,0) \in \mathbb{R}^2$ and $f(x,y)$ is a homogeneous polynomial of some degree $p_z \geq 2$ without multiple factors.

It follows from (2) that all critical points of f are isolated. Let $n = |\Sigma_f|$ be the total number of such points.

If $p_z = 2$ for all $z \in \Sigma_f$, then f is Morse and the calculations of the homotopy types are given in [**?**, **?**].

Theorem. *Suppose that* $f: M \to P$ *satisfies* (1) *and* (2) *and suppose that* $p_z \geq 3$ *for some* $z \in \Sigma_f$. *Then*

1) $\mathcal{S}(f)$ *and* $\mathcal{O}(f, \Sigma_f)$ *are contractible;*

2) $\mathcal{O}(f)$ *is homotopy equivalent to a CW-complex of dimension* $\leq 2n$, $\pi_i \mathcal{O}(f) = \pi_i(M)$ *for* $i \geq 3$, $\pi_2 \mathcal{O}(f) = 0$, *and we have the following exact sequence:*

$$1 \to \pi_1 \mathrm{Diff}_{\mathrm{id}}(M) \oplus \mathbb{Z}^k \to \pi_1 \mathcal{O}(f) \to G \to 1,$$

where G is a (finite) subgroup of the group of automorphisms of the Kronrod-Reeb graph of f and $k \geq 0$.

References

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