Deformation of Sasakian Metrics

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In this talk, we will discuss on homotopy invariance of geometrically tautness of 1-dimensional Riemannian foliations and a stability property of Sasakian metrics in families of 1-dimensional transversely holomorphic Riemannian foliations. The first one is the key part of the proof of the second one.

A foliated manifold $(M, \mathcal{F})$ is called geometrically taut if there exists a Riemannian metric g on M such that every leaf of $\mathcal{F}$ is a minimal submanifold of (M, g). If $(M, \mathcal{F})$ is Riemannian, this property was shown to be equivalent to the triviality of a basic cohomology class by Alvarez Lopez. We will discuss algebraicity of this cohomology class in the case that $\mathcal{F}$ is 1-dimensional and transversely parallelizable which implies homotopy invariance of geometrically tautness of 1-dimensional Riemannian foliations.

Sasakian metrics can be viewed as an odd dimensional version of Kähler metrics and have many similar properties to Kähler metrics. Although Kähler metrics have stability in families of compact complex manifolds due to a theorem of Kodaira and Spencer, Sasakian metrics were known to be not stable in many situation. We will discuss a characterization of stability of Sasakian metrics in families of 1-dimensional transversely holomorphic Riemannian foliations in terms of basic Euler classes of 1-dimensional Riemannian foliations.