On the structure of classical diffeomorphism groups

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The group of volume preserving diffeomorphisms, the group of symplectomorphisms and the group of contactomorphisms constitute the classical groups of diffeomorphisms. The first homology groups of the compactly supported identity components of the first two groups have been computed by Thurston and Banyaga, respectively. They showed that, in general, the first homology groups are not equal to zero, and that they can be expressed by the flux homomorphism, the Calabi homomorphism and other invariants. The results and methods of their proofs are similar in both cases.

In this talk we present a solution of the long standing problem on the algebraic structure of the third classical diffeomorphism group, i.e. the contactomorphism group. Namely we show that the compactly supported identity component of the group of contactomorphisms is perfect, i.e. its first homology group vanishes. By a theorem of Epstein it is simple as well. In the proof we use well-known facts (Schauder-Tichonoff fixed point theorem, Lychagin chart in the contactomorphism group) and new constructions (e.g. those of fragmentations of contactomorphisms of the second kind, or of a rolling-up operator). The proof is specially tailored for contactomorphisms and it cannot be carried over to other diffeomorphism groups. Possible applications to Haefliger's classifying spaces of contact foliations are indicated.