The dynamics of expansive homeomorphisms with the shadowing property may be very complicated but it is quite well understood (see Aoki and Hiraide’s monograph, for example). It is known that these systems admit only a finite number of chain recurrent classes (Spectral Decomposition Theorem). In 2012, Morales introduced a generalization of expansivity property, called $N$-expansive property. For every $N \in \mathbb{N}$, we will exhibit an $N$-expansive homeomorphism, which is not $(N-1)$-expansive, has the shadowing property, and admits an infinite number of chain-recurrent classes. We discuss some properties of the local stable (unstable) sets of $N$-expansive homeomorphisms with the shadowing property and use them to prove that some types of the limit shadowing property are present. We will discuss a Spectral Decomposition Theorem for $N$-expansive homeomorphisms with the shadowing property defined on surfaces.