Summary of the PhD thesis
„Locally finite polynomial endomorphisms"

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Locally finite polynomial endomorphism are defined to be polynomial mappings $F : \mathbb{C}^n \to \mathbb{C}^n$ for which the equation

$$p(F) := \sum_{i=0}^{d} p_i F^{\circ i} = 0, \quad \text{where } F^{\circ i} := \underbrace{F \circ \ldots \circ F}_{i \text{ times}}$$

holds true for some nonzero polynomial $p = \sum_{i=0}^{d} p_i T^i \in \mathbb{C}[T]$. Any polynomial p that satisfies the above equation is called the characteristic polynomial for F; characteristic polynomial p that is monic (i. e. $p_d = 1$) and of the lowest degree is called the minimal polynomial for F.

In the first part of the thesis most properties of locally finite endomorphisms - proved by Furter and Maubach in the complex case - are shown to be valid for polynomial maps defined over an arbitrary field of characteristic 0.

Then, improving existing results on the subject, a formula for characteristic polynomial depending on the linear part of F is given (Theorem 2.3). The minimal polynomial for a subclass of triangular automorphisms is explicitly found in Theorem 2.14 and in the case $n = 2$ an optimal estimate for the degree of the minimal polynomial is presented (Theorem 2.18).

Next, normal subgroups of $\mathrm{GA}_n(\mathbf{k})$ - the polynomial automorphisms group of $\mathbf{k}^n$ - are considered. It is shown, that for any field k of characteristic 0 the subgroup of $\mathrm{GA}_n(\mathbf{k})$ generated by all locally finite automorphisms is normal; moreover a conjecture about it being equal to $\mathrm{GA}_n(\mathbf{k})$ is stated.

Finally, the interplay between locally finite automorphisms and derivations on the ring of polynomials is studied. In Theorem 4.4 an exact formula for the minimal polynomial of an exponential automorphism F (i.e. $F = \exp(D)$, where D is a locally nilpotent derivation) is found. Conversely, given a triangular automorphism F of $\mathbf{k}^n$, locally finite derivation D such that $F = \exp(D)$ is also calculated (Theorem 4.9).