1. Using L'Hospital's rule (if necessary), calculate the following limits:

a) \( \lim_{x \to 0} \frac{e^x - 1}{\sin 2x} \);  
b) \( \lim_{x \to 0} \frac{1 - x + \ln x}{\cos(\pi x)} \);  
c) \( \lim_{x \to 0} \frac{\sin x}{x + x^2} \);  
d) \( \lim_{x \to +\infty} \frac{e^x}{x + x^2} \);  
e) \( \lim_{x \to +\infty} \frac{\ln x}{\sqrt{x}} \);  
f) \( \lim_{x \to +\infty} \left( \frac{1}{x} - \frac{1}{\sin x} \right) \);  
g) \( \lim_{x \to 0} \frac{1}{x} - \cot x \);  
h) \( \lim_{x \to 0} \frac{1}{x - 2} - \frac{1}{\ln(x - 1)} \);  
i) \( \lim_{x \to +\infty} \sqrt{x^2 + 3x - x} \);  
j) \( \lim_{x \to 0} x \ln x \);  
k) \( \lim_{x \to 0} (\cos x)^{\frac{3}{2}} \);  
l) \( \lim_{x \to +\infty} x \left( \left( 1 + \frac{1}{x} \right)^x - e \right) \).


a) Let \( D \) be a demand function varying inversely proportional to the price \( p \). If the \( p \) increases by one percent, what is the percentage change in demand.

b) Assume that, as a function of the price \( p \), the demand for chocolate from the the rural population is three times smaller than the demand from the urban population with elasticity \( -0.8 \) in the rural case, and elasticity 0.3 in the urban case. Approximately what percentage change in global demand for chocolate will result from an increase in the price by 1%?

c) Let \( C(q) \) denote the cost of producing \( q \) widgets, and \( c(q) = \frac{C(q)}{q} \) the average cost of producing one widget. Assume that \( C \) extends to a differentiable function of \( (0, \infty) \) and show that \( C'(q) = c(q)(1 + E_q(c)) \).

3. Calculate the Taylor series together with its radius of convergence for the following functions:

a) \( f(x) = \frac{1}{1 + x}, \ x_0 = 0; \)

b) \( f(x) = \frac{1}{2 + x^2}, \ x_0 = 0; \)

c) \( f(x) = \frac{1 + x}{2 + 3x^2}, \ x_0 = 0; \)

d) \( f(x) = \frac{1}{(x - 2)(4 - x)}; \ x_0 = 3; \)

e) \( f(x) = \frac{1}{(x - 2)(4 - x)}, \ x_0 = \frac{5}{2}; \)

f) \( f(x) = \frac{1}{(x - 4)^2}, \ x_0 = 5; \)

g) \( f(x) = (1 + x)^2, \ x_0 = 2; \)

h) \( f(x) = (1 + x)^{\frac{1}{2}}, \ x_0 = 0 \)

i) \( f(x) = (1 - x)^{\frac{1}{3}}, \ x_0 = 0; \)

j) \( f(x) = (1 - x)^{-\frac{1}{3}}, \ x_0 = 0; \)

k) \( f(x) = \sin(x^2), \ x_0 = 0; \)

l) \( f(x) = \sin^2(x), \ x_0 = 0. \)

4. Use the Taylor formula together with the Lagrange formula for the error to calculate:

a) The number \( e \) with 5 decimal places of accuracy.

b) The number \( \sqrt{105} \) with 3 decimal places of accuracy.

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