0. Find „left–side” and „right–side” limits of functions:
   a) \( f(x) = \frac{2^{1/x} - 3}{3^{1/x} + 2} \), at point \( x = 1 \),
   b) \( f(x) = \frac{2^{1/x} + 3}{3^{1/x} + 2} \), at point \( x = 0 \).

1. Find functions \( f(g(x)) \) and \( g(f(x)) \), where:
   a) \( f(x) = 1 - x^2 \), \( g(x) = 2x + 3 \);
   b) \( f(x) = -17 \), \( g(x) = |x| \);
   c) \( f(x) = \sqrt{x^2 - 3} \), \( g(x) = x^2 + 3 \);
   d) \( f(x) = x^2 + 1 \), \( g(x) = \frac{1}{x^{x+1}} \);
   e) \( f(x) = x^3 - 4 \), \( g(x) = \frac{1}{\sqrt[3]{x} + 4} \).

2. Find a function of the form \( f(x) = x^k \) (\( k \) does not have to be an integer) and a function \( g(x) \) in such a way that \( f(g(x)) = h(x) \), where:
   a) \( h(x) = \frac{1}{1 + x^2} \);
   b) \( h(x) = \frac{1}{\sqrt{x + 10}} \);
   c) \( h(x) = \frac{1}{(1 + x + x^2)^3} \).

3. Let \( f(x) = 1 + x^2 \). Find a function \( g(x) \), to have \( f(g(x)) = 1 + x^2 - 2x^3 + x^4 \).

4. Let \( g(x) = 1 + \sqrt{x} \). Find a function \( f(x) \), to have \( f(g(x)) = 3 + 2\sqrt{x} + x \).

5. Find a function \( g(x) \) to have \( f(g(x)) = h(x) \), where \( f(x) = x^2 \), \( h(x) = x^4 + 1 \).

6. a) Is this possible to choose the value \( f(1) \), in such a way that the function defined for \( x \neq 1 \) by the formula
   \[ f(x) = \frac{|x - 1|}{(x - 1)^3} \]
   is continuous on \( \mathbb{R} \)?
   b) Is this possible to choose the values \( f(-2), f(3) \), in such a way that the function defined for \( x \in \mathbb{R} \setminus \{-2, 3\} \) as
   \[ f(x) = \frac{x + 1}{x^2 - x - 6} \]
   is continuous on \( \mathbb{R} \)?
   c) Is this possible to choose the values \( f(-1), f(1) \), in such a way that the function defined for \( x \in \mathbb{R} \setminus \{-1, 1\} \) as
   \[ f(x) = \frac{|x^2 - 1|}{x^2 - 1} \]
   is continuous on \( \mathbb{R} \)?
7. a) Show that equation $x^4 + 2x - 1 = 0$ is satisfied for some $x \in [0, 1]$.
    b) Show that equation $x^5 - 5x^3 + 3 = 0$ is satisfied for some $x \in [-3, 2]$.
    c) Show that equation $x^3 - 4x + 1 = 0$ has three different real valued solutions.
    d) Show that there is an $x$ between $\frac{\pi}{2}$ and $\pi$ such that $\tan x = -x$.

8. At which points the following function is continuous

$$f(x) = \begin{cases} 
0, & x \in \mathbb{Q}, \\
x^2, & x \notin \mathbb{Q}
\end{cases}$$

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